F01QGF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

F01QGF reduces the m by n $(m \le n)$ real upper trapezoidal matrix A to upper triangular form by means of orthogonal transformations.

2 Specification

```
SUBROUTINE F01QGF(M, N, A, LDA, ZETA, IFAIL)
INTEGER           M, N, LDA, IFAIL
real              A(LDA,*), ZETA(*)
```

3 Description

The m by n $(m \le n)$ real upper trapezoidal matrix A given by

$$A = (U\ X),$$

where U is an m by m upper triangular matrix, is factorized as

$$A = (R\ 0)P^T,$$

where P is an n by n orthogonal matrix and R is an m by m upper triangular matrix.

P is given as a sequence of Householder transformation matrices

$$P = P_m \dots P_2 P_1,$$

the $(m-k+1)$th transformation matrix, P_k, being used to introduce zeros into the kth row of A. P_k has the form

$$P_k = \begin{pmatrix} I & 0 \\ 0 & T_k \end{pmatrix},$$

where

$$\mathrm{T}_k = I - u_k u_k^T,$$
$$\mathrm{u}_k = \begin{pmatrix} \zeta_k \\ 0 \\ z_k \end{pmatrix},$$

ζ_k is a scalar and z_k is an $(n-m)$ element vector. ζ_k and z_k are chosen to annihilate the elements of the kth row of X.

The vector u_k is returned in the kth element of the array ZETA and in the kth row of A, such that ζ_k is in ZETA(k) and the elements of z_k are in A$(k, m+1)$,. . .,A(k, n). The elements of R are returned in the upper triangular part of A.

For further information on this factorization and its use see Section 6.5 of Golub and Van Loan [1].

4 References

[1] Golub G H and Van Loan C F (1989) *Matrix Computations* Johns Hopkins University Press (2nd Edition), Baltimore

[2] Wilkinson J H (1965) *The Algebraic Eigenvalue Problem* Oxford University Press, London

5 Parameters

1: M — INTEGER *Input*

On entry: m, the number of rows of A.

When M = 0 then an immediate return is effected.

Constraint: M $\geq$ 0.

2: N — INTEGER *Input*

On entry: n, the number of columns of A.

Constraint: N $\geq$ M.

3: A(LDA,$*$) — ***real*** array *Input/Output*

Note: the second dimension of the array A must be at least max(1,N).

On entry: the leading m by n upper trapezoidal part of the array A must contain the matrix to be factorized.

On exit: the m by m upper triangular part of A will contain the upper triangular matrix R, and the m by $(n-m)$ upper trapezoidal part of A will contain details of the factorization as described in Section 3.

4: LDA — INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F01QGF is called.

Constraint: LDA $\geq$ max(1,M).

5: ZETA($*$) — ***real*** array *Output*

Note: the dimension of the array ZETA must be at least max(1,M).

On exit: ZETA(k) contains the scalar ζ_k for the $(m-k+1)$th transformation. If $T_k = I$ then ZETA(k) = 0.0, otherwise ZETA(k) contains ζ_k as described in Section 3 and ζ_k is always in the range $(1.0, \sqrt{2.0})$.

6: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors detected by the routine:

IFAIL = -1

On entry, M < 0,
or N < M,
or LDA < M.

7 Accuracy

The computed factors R and P satisfy the relation

$$(R\ 0)P^T = A + E,$$

where

$$\|E\| \leq c\epsilon\|A\|,$$

ϵ is the ***machine precision*** (see X02AJF), c is a modest function of m and n and $\|.\|$ denotes the spectral (two) norm.

8 Further Comments

The approximate number of floating-point operations is given by $2m^2(n-m)$.

9 Example

To reduce the 3 by 5 matrix

$$A = \begin{pmatrix} 2.4 & 0.8 & -1.4 & 3.0 & -0.8 \\ 0.0 & 1.6 & 0.8 & 0.4 & -0.8 \\ 0.0 & 0.0 & 1.0 & 2.0 & 2.0 \end{pmatrix}$$

to upper triangular form.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     F01QGF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          MMAX, NMAX, LDA
      PARAMETER        (MMAX=10,NMAX=20,LDA=MMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, J, M, N
*     .. Local Arrays ..
      real             A(LDA,NMAX), ZETA(MMAX)
*     .. External Subroutines ..
      EXTERNAL         F01QGF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F01QGF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) M, N
      WRITE (NOUT,*)
      IF ((M.GT.MMAX) .OR. (N.GT.NMAX)) THEN
         WRITE (NOUT,*) 'M or N is out of range.'
         WRITE (NOUT,99999) 'M = ', M, '   N = ', N
      ELSE
         READ (NIN,*) ((A(I,J),J=1,N),I=1,M)
         IFAIL = 0
*
*        Find the RQ factorization of A
         CALL F01QGF(M,N,A,LDA,ZETA,IFAIL)
*
```

```
         WRITE (NOUT,*) 'RQ factorization of A'
         WRITE (NOUT,*)
         WRITE (NOUT,*) 'Vector ZETA'
         WRITE (NOUT,99998) (ZETA(I),I=1,M)
         WRITE (NOUT,*)
         WRITE (NOUT,*)
     + 'Matrix A after factorization (R is in left-hand upper triangle)'
         DO 20 I = 1, M
            WRITE (NOUT,99998) (A(I,J),J=1,N)
   20    CONTINUE
      END IF
      STOP
*
99999 FORMAT (1X,A,I5,A,I5)
99998 FORMAT (5(1X,F8.4))
      END
```

9.2 Program Data

```
F01QGF Example Program Data
  3    5                        :Values of M and N
  2.4   0.8  -1.4   3.0  -0.8
   0    1.6   0.8   0.4  -0.8
   0     0    1.0   2.0   2.0   :End of matrix A
```

9.3 Program Results

```
 F01QGF Example Program Results

 RQ factorization of A

 Vector ZETA
   1.2649   1.3416   1.1547

 Matrix A after factorization (R is in left-hand upper triangle)
  -4.0000  -1.0000  -1.0000   0.6325   0.0000
   0.0000  -2.0000   0.0000   0.0000  -0.4472
   0.0000   0.0000  -3.0000   0.5774   0.5774
```