F07FDF (SPOTRF/DPOTRF) – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

F07FDF (SPOTRF/DPOTRF) computes the Cholesky factorization of a real symmetric positive-definite matrix.

2 Specification

```
SUBROUTINE F07FDF(UPLO, N, A, LDA, INFO)
ENTRY      spotrf(UPLO, N, A, LDA, INFO)
INTEGER            N, LDA, INFO
real               A(LDA,*)
CHARACTER*1        UPLO
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

This routine forms the Cholesky factorization of a real symmetric positive-definite matrix A either as $A = U^T U$ if UPLO = 'U', or $A = LL^T$ if UPLO = 'L', where U is an upper triangular matrix and L is lower triangular.

4 References

[1] Demmel J W (1989) On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville

[2] Golub G H and van Loan C F (1996) *Matrix Computations* Johns Hopkins University Press (3rd Edition), Baltimore

5 Parameters

1: UPLO — CHARACTER*1 *Input*

On entry: indicates whether the upper or lower triangular part of A is stored and how A is factorized, as follows:

if UPLO = 'U', then the upper triangular part of A is stored and A is factorized as $U^T U$, where U is upper triangular;
if UPLO = 'L', then the lower triangular part of A is stored and A is factorized as LL^T, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N — INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

3: A(LDA,∗) — ***real*** array *Input/Output*

Note: the second dimension of the array A must be at least max(1,N).

On entry: the n by n symmetric positive-definite matrix A. If UPLO = 'U', the upper triangle of A must be stored and the elements of the array below the diagonal are not referenced; if UPLO = 'L', the lower triangle of A must be stored and the elements of the array above the diagonal are not referenced.

On exit: the upper or lower triangle of A is overwritten by the Cholesky factor U or L as specified by UPLO.

4: LDA — INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07FDF (SPOTRF/DPOTRF) is called.

Constraint: LDA $\geq$ max(1,N).

5: INFO — INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, the leading minor of order i is not positive-definite and the factorization could not be completed. Hence A itself is not positive-definite. This may indicate an error in forming the matrix A. To factorize a symmetric matrix which is not positive-definite, call F07MDF (SSYTRF/DSYTRF) instead.

7 Accuracy

If UPLO = 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon |U^T||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***. If UPLO = 'L', a similar statement holds for the computed factor L. It follows that $|e_{ij}| \leq c(n)\epsilon \sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of floating-point operations is approximately $\frac{1}{3}n^3$.

A call to this routine may be followed by calls to the routines:

F07FEF (SPOTRS/DPOTRS) to solve $AX = B$;

F07FGF (SPOCON/DPOCON) to estimate the condition number of A;

F07FJF (SPOTRI/DPOTRI) to compute the inverse of A.

The complex analogue of this routine is F07FRF (CPOTRF/ZPOTRF).

9 Example

To compute the Cholesky factorization of the matrix A, where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix}.$$

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     F07FDF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, LDA
      PARAMETER        (NMAX=8,LDA=NMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
      CHARACTER        UPLO
*     .. Local Arrays ..
      real             A(LDA,NMAX)
*     .. External Subroutines ..
      EXTERNAL         spotrf, X04CAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07FDF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read A from data file
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            READ (NIN,*) ((A(I,J),J=I,N),I=1,N)
         ELSE IF (UPLO.EQ.'L') THEN
            READ (NIN,*) ((A(I,J),J=1,I),I=1,N)
         END IF
*
*        Factorize A
*
         CALL spotrf(UPLO,N,A,LDA,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.EQ.0) THEN
*
*           Print factor
*
            IFAIL = 0
            CALL X04CAF(UPLO,'Nonunit',N,N,A,LDA,'Factor',IFAIL)
         ELSE
            WRITE (NOUT,*) 'A is not positive-definite'
         END IF
      END IF
      STOP
*
      END
```

9.2 Program Data

```
F07FDF Example Program Data
  4                           :Value of N
  'L'                         :Value of UPLO
  4.16
 -3.12   5.03
  0.56  -0.83   0.76
 -0.10   1.18   0.34   1.18   :End of matrix A
```

9.3 Program Results

```
F07FDF Example Program Results

Factor
            1          2          3          4
1      2.0396
2     -1.5297     1.6401
3      0.2746    -0.2500     0.7887
4     -0.0490     0.6737     0.6617     0.5347
```