NAG Fortran Library Routine Document

F08GUF (CUPMTR/ZUPMTR)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08GUF (CUPMTR/ZUPMTR) multiplies an arbitrary complex matrix C by the complex unitary matrix Q which was determined by F08GSF (CHPTRD/ZHPTRD) when reducing a complex Hermitian matrix to tridiagonal form.

2 Specification

```
SUBROUTINE F08GUF(SIDE, UPLO, TRANS, M, N, AP, TAU, C, LDC, WORK, INFO)
ENTRY      cupmtr (SIDE, UPLO, TRANS, M, N, AP, TAU, C, LDC, WORK, INFO)
INTEGER             M, N, LDC, INFO
complex             AP(*), TAU(*), C(LDC,*), WORK(*)
CHARACTER*1         SIDE, UPLO, TRANS
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

This routine is intended to be used after a call to F08GSF (CHPTRD/ZHPTRD), which reduces a complex Hermitian matrix A to real symmetric tridiagonal form T by a unitary similarity transformation: $A = QTQ^H$. F08GSF represents the unitary matrix Q as a product of elementary reflectors.

This routine may be used to form one of the matrix products

$$QC,\ Q^HC,\ CQ \text{ or } CQ^H,$$

overwriting the result on C (which may be any complex rectangular matrix).

A common application of this routine is to transform a matrix Z of eigenvectors of T to the matrix QZ of eigenvectors of A.

4 References

Golub G H and van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: SIDE – CHARACTER*1 *Input*

On entry: indicates how Q or Q^H is to be applied to C as follows:

if SIDE = 'L', Q or Q^H is applied to C from the left;

if SIDE = 'R', Q or Q^H is applied to C from the right.

Constraint: SIDE = 'L' or 'R'.

2: UPLO – CHARACTER*1 *Input*

On entry: this **must** be the same parameter UPLO as supplied to F08GSF (CHPTRD/ZHPTRD).

Constraint: UPLO = 'U' or 'L'.

3: TRANS – CHARACTER*1 *Input*

On entry: indicates whether Q or Q^H is to be applied to C as follows:

if TRANS = 'N', Q is applied to C;

if TRANS = 'C', Q^H is applied to C.

Constraint: TRANS = 'N' or 'C'.

4: M – INTEGER *Input*

On entry: m, the number of rows of the matrix C; m is also the order of Q if SIDE = 'L'.

Constraint: M ≥ 0.

5: N – INTEGER *Input*

On entry: n, the number of columns of the matrix C; n is also the order of Q if SIDE = 'R'.

Constraint: N ≥ 0.

6: AP(*) – ***complex*** array *Input/Output*

Note: the dimension of the array AP must be at least $\max(1, \mathrm{M} * (\mathrm{M}+1)/2)$ if SIDE = 'L' and at least $\max(1, \mathrm{N} * (\mathrm{N}+1)/2)$ if SIDE = 'R' .

On entry: details of the vectors which define the elementary reflectors, as returned by F08GSF (CHPTRD/ZHPTRD).

On exit: AP is used as internal workspace prior to being restored and hence is unchanged.

7: TAU(*) – ***complex*** array *Input*

Note: the dimension of the array TAU must be at least $\max(1, \mathrm{M}-1)$ if SIDE = 'L' and at least $\max(1, \mathrm{N}-1)$ if SIDE = 'R' .

On entry: further details of the elementary reflectors, as returned by F08GSF (CHPTRD/ZHPTRD).

8: C(LDC,*) – ***complex*** array *Input/Output*

Note: the second dimension of the array C must be at least $\max(1, \mathrm{N})$.

On entry: the m by n matrix C.

On exit: C is overwritten by QC or Q^HC or CQ or CQ^H as specified by SIDE and TRANS.

9: LDC – INTEGER *Input*

On entry: the first dimension of the array C as declared in the (sub)program from which F08GUF (CUPMTR/ZUPMTR) is called.

Constraint: LDC $\geq \max(1, \mathrm{M})$.

10: WORK(*) – ***complex*** array *Workspace*

Note: the dimension of the array WORK must be at least $\max(1, \mathrm{N})$ if SIDE = 'L' and at least $\max(1, \mathrm{M})$ if SIDE = 'R'.

11: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed result differs from the exact result by a matrix E such that

$$\|E\|_2 = O(\epsilon)\|C\|_2,$$

where ϵ is the ***machine precision***.

8 Further Comments

The total number of real floating-point operations is approximately $8m^2n$ if SIDE = 'L' and $8mn^2$ if SIDE = 'R'.

The real analogue of this routine is F08GGF (SOPMTR/DOPMTR).

9 Example

To compute the two smallest eigenvalues, and the associated eigenvectors, of the matrix A, where

$$A = \begin{pmatrix} -2.28+0.00i & 1.78-2.03i & 2.26+0.10i & -0.12+2.53i \\ 1.78+2.03i & -1.12+0.00i & 0.01+0.43i & -1.07+0.86i \\ 2.26-0.10i & 0.01-0.43i & -0.37+0.00i & 2.31-0.92i \\ -0.12-2.53i & -1.07-0.86i & 2.31+0.92i & -0.73+0.00i \end{pmatrix},$$

using packed storage. Here A is Hermitian and must first be reduced to tridiagonal form T by F08GSF (CHPTRD/ZHPTRD). The program then calls F08JJF (SSTEBZ/DSTEBZ) to compute the requested eigenvalues and F08JXF (CSTEIN/ZSTEIN) to compute the associated eigenvectors of T. Finally F08GUF (CUPMTR/ZUPMTR) is called to transform the eigenvectors to those of A.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     F08GUF Example Program Text
*     Mark 16 Release. NAG Copyright 1992.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, LDZ
      PARAMETER        (NMAX=8,LDZ=NMAX)
      real             ZERO
      PARAMETER        (ZERO=0.0e0)
*     .. Local Scalars ..
      real             VL, VU
      INTEGER          I, IFAIL, INFO, J, M, N, NSPLIT
      CHARACTER        UPLO
*     .. Local Arrays ..
      complex          AP(NMAX*(NMAX+1)/2), TAU(NMAX), WORK(NMAX),
     +                 Z(LDZ,NMAX)
      real             D(NMAX), E(NMAX), RWORK(5*NMAX), W(NMAX)
      INTEGER          IBLOCK(NMAX), IFAILV(NMAX), ISPLIT(NMAX),
     +                 IWORK(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*     .. External Subroutines ..
```

```
      EXTERNAL         sstebz, X04DBF, chptrd, cstein, cupmtr
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F08GUF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read A from data file
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            READ (NIN,*) ((AP(I+J*(J-1)/2),J=I,N),I=1,N)
         ELSE IF (UPLO.EQ.'L') THEN
            READ (NIN,*) ((AP(I+(2*N-J)*(J-1)/2),J=1,I),I=1,N)
         END IF
*
*        Reduce A to tridiagonal form T = (Q**H)*A*Q
*
         CALL chptrd(UPLO,N,AP,D,E,TAU,INFO)
*
*        Calculate the two smallest eigenvalues of T (same as A)
*
         CALL sstebz('I','B',N,VL,VU,1,2,ZERO,D,E,M,NSPLIT,W,IBLOCK,
     +               ISPLIT,RWORK,IWORK,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.GT.0) THEN
            WRITE (NOUT,*) 'Failure to converge.'
         ELSE
            WRITE (NOUT,*) 'Eigenvalues'
            WRITE (NOUT,99999) (W(I),I=1,M)
*
*           Calculate the eigenvectors of T, storing the result in Z
*
            CALL cstein(N,D,E,M,W,IBLOCK,ISPLIT,Z,LDZ,RWORK,IWORK,
     +                  IFAILV,INFO)
*
            IF (INFO.GT.0) THEN
               WRITE (NOUT,*) 'Failure to converge.'
            ELSE
*
*              Calculate the eigenvectors of A = Q * (eigenvectors of T)
*
               CALL cupmtr('Left',UPLO,'No transpose',N,M,AP,TAU,Z,LDZ,
     +                     WORK,INFO)
*
*              Print eigenvectors
*
               WRITE (NOUT,*)
               IFAIL = 0
*
               CALL X04DBF('General',' ',N,M,Z,LDZ,'Bracketed','F7.4',
     +                     'Eigenvectors','Integer',RLABS,'Integer',
     +                     CLABS,80,0,IFAIL)
*
            END IF
         END IF
      END IF
      STOP
*
99999 FORMAT (8X,4(F7.4,11X,:))
      END
```

9.2 Program Data

```
F08GUF Example Program Data
  4                                                                :Value of N
  'L'                                                              :Value of UPLO
 (-2.28, 0.00)
 ( 1.78, 2.03) (-1.12, 0.00)
 ( 2.26,-0.10) ( 0.01,-0.43) (-0.37, 0.00)
 (-0.12,-2.53) (-1.07,-0.86) ( 2.31, 0.92) (-0.73, 0.00)   :End of matrix A
```

9.3 Program Results

```
 F08GUF Example Program Results

 Eigenvalues
       -6.0002          -3.0030

 Eigenvectors
                  1                 2
 1  ( 0.7299, 0.0000) (-0.2595, 0.0000)
 2  (-0.1663,-0.2061) ( 0.5969, 0.4214)
 3  (-0.4165,-0.1417) (-0.2965,-0.1507)
 4  ( 0.1743, 0.4162) ( 0.3482, 0.4085)
```