

# NAG Fortran Library Routine Document

## F08NFF (SORGHR/DORGHR)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

F08NFF (SORGHR/DORGHR) generates the real orthogonal matrix  $Q$  which was determined by F08NEF (SGEHRD/DGEHRD) when reducing a real general matrix  $A$  to Hessenberg form.

### 2 Specification

```
SUBROUTINE F08NFF(N, ILO, IHI, A, LDA, TAU, WORK, LWORK, INFO)
ENTRY      sorghr (N, ILO, IHI, A, LDA, TAU, WORK, LWORK, INFO)
INTEGER    N, ILO, IHI, LDA, LWORK, INFO
real     A(LDA,*), TAU(*), WORK(*)
```

The ENTRY statement enables the routine to be called by its LAPACK name.

### 3 Description

This routine is intended to be used following a call to F08NEF (SGEHRD/DGEHRD), which reduces a real general matrix  $A$  to upper Hessenberg form  $H$  by an orthogonal similarity transformation:  $A = QHQ^T$ . F08NEF (SGEHRD/DGEHRD) represents the matrix  $Q$  as a product of  $i_{hi} - i_{lo}$  elementary reflectors. Here  $i_{lo}$  and  $i_{hi}$  are values determined by F08NHF (SGEBAL/DGEBAL) when balancing the matrix; if the matrix has not been balanced,  $i_{lo} = 1$  and  $i_{hi} = n$ .

This routine may be used to generate  $Q$  explicitly as a square matrix.  $Q$  has the structure:

$$Q = \begin{pmatrix} I & 0 & 0 \\ 0 & Q_{22} & 0 \\ 0 & 0 & I \end{pmatrix}$$

where  $Q_{22}$  occupies rows and columns  $i_{lo}$  to  $i_{hi}$ .

### 4 References

Golub G H and van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

### 5 Parameters

1: N – INTEGER *Input*

*On entry:*  $n$ , the order of the matrix  $Q$ .

*Constraint:*  $N \geq 0$ .

2: ILO – INTEGER *Input*

3: IHI – INTEGER *Input*

*On entry:* these **must** be the same parameters ILO and IHI, respectively, as supplied to F08NEF (SGEHRD/DGEHRD).

*Constraints:*

$$\begin{aligned} 1 &\leq \text{ILO} \leq \text{IHI} \leq N \text{ if } N > 0, \\ \text{ILO} &= 1 \text{ and } \text{IHI} = 0 \text{ if } N = 0. \end{aligned}$$

- 4: A(LDA,\*) – *real* array Input/Output  
**Note:** the second dimension of the array A must be at least  $\max(1, N)$ .  
*On entry:* details of the vectors which define the elementary reflectors, as returned by F08NEF (SGEHRD/DGEHRD).  
*On exit:* the  $n$  by  $n$  orthogonal matrix  $Q$ .
- 5: LDA – INTEGER Input  
*On entry:* the first dimension of the array A as declared in the (sub)program from which F08NFF (SORGHR/DORGHR) is called.  
*Constraint:*  $LDA \geq \max(1, N)$ .
- 6: TAU(\*) – *real* array Input  
**Note:** the dimension of the array TAU must be at least  $\max(1, N - 1)$ .  
*On entry:* further details of the elementary reflectors, as returned by F08NEF (SGEHRD/DGEHRD).
- 7: WORK(\*) – *real* array Workspace  
**Note:** the dimension of the array WORK must be at least  $\max(1, LWORK)$ .  
*On exit:* if  $INFO = 0$ ,  $WORK(1)$  contains the minimum value of  $LWORK$  required for optimum performance.
- 8: LWORK – INTEGER Input  
*On entry:* the dimension of the array WORK as declared in the (sub)program from which F08NFF (SORGHR/DORGHR) is called, unless  $LWORK = -1$ , in which case a workspace query is assumed and the routine only calculates the optimal dimension of WORK (using the formula given below).  
*Suggested value:* for optimum performance  $LWORK$  should be at least  $(IHI - ILO) \times nb$ , where  $nb$  is the *blocksize*.  
*Constraint:*  $LWORK \geq \max(1, IHI - ILO)$  or  $LWORK = -1$ .
- 9: INFO – INTEGER Output  
*On exit:*  $INFO = 0$  unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the routine:

$INFO < 0$

If  $INFO = -i$ , the  $i$ th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

## 7 Accuracy

The computed matrix  $Q$  differs from an exactly orthogonal matrix by a matrix  $E$  such that

$$\|E\|_2 = O(\epsilon),$$

where  $\epsilon$  is the *machine precision*.

## 8 Further Comments

The total number of floating-point operations is approximately  $\frac{4}{3}q^3$ , where  $q = i_{hi} - i_{lo}$ .

The complex analogue of this routine is F08NTF (CUNGHR/ZUNGHR).

## 9 Example

To compute the Schur factorization of the matrix  $A$ , where

$$A = \begin{pmatrix} 0.35 & 0.45 & -0.14 & -0.17 \\ 0.09 & 0.07 & -0.54 & 0.35 \\ -0.44 & -0.33 & -0.03 & 0.17 \\ 0.25 & -0.32 & -0.13 & 0.11 \end{pmatrix}.$$

Here  $A$  is general and must first be reduced to Hessenberg form by F08NEF (SGEHRD/DGEHRD). The program then calls F08NFF (SORGHR/DORGHR) to form  $Q$ , and passes this matrix to F08PEF (SHSEQR/DHSEQR) which computes the Schur factorization of  $A$ .

### 9.1 Program Text

**Note:** the listing of the example program presented below uses *bold italicised* terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      F08NFF Example Program Text
*      Mark 16 Release. NAG Copyright 1992.
*      .. Parameters ..
INTEGER          NIN, NOUT
PARAMETER       (NIN=5,NOUT=6)
INTEGER          NMAX, LDA, LDZ, LWORK
PARAMETER       (NMAX=8,LDA=NMAX,LDZ=NMAX,LWORK=64*(NMAX-1))
*      .. Local Scalars ..
INTEGER          I, IFAIL, INFO, J, N
*      .. Local Arrays ..
real           A(LDA,NMAX), TAU(NMAX), WI(NMAX), WORK(LWORK),
+              WR(NMAX), Z(LDZ,NMAX)
*      .. External Subroutines ..
EXTERNAL        sgehrd, shseqr, sorghr, F06QFF, X04CAF
*      .. Executable Statements ..
WRITE (NOUT,*) 'F08NFF Example Program Results'
Skip heading in data file
READ (NIN,*)
READ (NIN,*) N
IF (N.LE.NMAX) THEN
*
*      Read A from data file
*
READ (NIN,*) ((A(I,J),J=1,N),I=1,N)
*
*      Reduce A to upper Hessenberg form H = (Q**T)*A*Q
*
CALL sgehrd(N,1,N,A,LDA,TAU,WORK,LWORK,INFO)
*
*      Copy A into Z
*
CALL F06QFF('General',N,N,A,LDA,Z,LDZ)
*
*      Form Q explicitly, storing the result in Z
*
CALL sorghr(N,1,N,Z,LDZ,TAU,WORK,LWORK,INFO)
*
*      Calculate the Schur factorization of H = Y*T*(Y**T) and form
*      Q*Y explicitly, storing the result in Z
*
*      Note that A = Z*T*(Z**T), where Z = Q*Y
*
CALL shseqr('Schur form','Vectors',N,1,N,A,LDA,WR,WI,Z,LDZ,
```

```

+          WORK,LWORK,INFO)
*
*   Print Schur form
*
*   WRITE (NOUT,*)
*   IFAIL = 0
*
*   CALL X04CAF('General',' ',N,N,A,LDA,'Schur form',IFAIL)
*
*   Print Schur vectors
*
*   WRITE (NOUT,*)
*   IFAIL = 0
*
*   CALL X04CAF('General',' ',N,N,Z,LDZ,'Schur vectors of A',IFAIL)
*
*   END IF
*   STOP
*
*   END

```

## 9.2 Program Data

F08NFF Example Program Data

```

4          :Value of N
0.35     0.45  -0.14  -0.17
0.09     0.07  -0.54   0.35
-0.44   -0.33  -0.03   0.17
0.25   -0.32  -0.13   0.11  :End of matrix A

```

## 9.3 Program Results

F08NFF Example Program Results

Schur form

	1	2	3	4
1	0.7995	0.0060	-0.1144	-0.0336
2	0.0000	-0.0994	-0.6483	-0.2026
3	0.0000	0.2478	-0.0994	-0.3474
4	0.0000	0.0000	0.0000	-0.1007

Schur vectors of A

	1	2	3	4
1	-0.6551	-0.3450	-0.1037	0.6641
2	-0.5236	0.6141	0.5807	-0.1068
3	0.5362	0.2935	0.3073	0.7293
4	-0.0956	0.6463	-0.7467	0.1249

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