NAG Fortran Library Routine Document

F08WEF (SGGHRD/DGGHRD)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1   Purpose

F08WEF (SGGHRD/DGGHRD) reduces a pair of real matrices (A, B), where B is upper triangular, to the generalized upper Hessenberg form using orthogonal transformations.

2   Specification

```
      SUBROUTINE F08WEF(COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q, LDQ, Z,
     1                  LDZ, INFO)
      ENTRY     sgghrd (COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q, LDQ, Z,
     1                  LDZ, INFO)
      INTEGER           N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
      real              A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
      CHARACTER*1       COMPQ, COMPZ
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3   Description

F08WEF (SGGHRD/DGGHRD) is the third step in the solution of the real generalized eigenvalue problem

$$Ax = \lambda Bx.$$

The (optional) first step balances the two matrices using F08WHF (SGGBAL/DGGBAL). In the second step, matrix B is reduced to upper triangular form using the QR factorization routine F08AEF (SGEQRF/DGEQRF) and this orthogonal transformation Q is applied to matrix A by calling F08AGF (SORMQR/DORMQR).

F08WEF (SGGHRD/DGGHRD) reduces a pair of real matrices (A, B), where B is upper triangular, to the generalized upper Hessenberg form using orthogonal transformations. This two-sided transformation is of the form

$$Q^T A Z = H$$
$$Q^T B Z = T$$

where H is an upper Hessenberg matrix, T is an upper triangular matrix and Q and Z are orthogonal, matrices determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q_1 and Z_1, so that

$$Q_1 A Z_1^T = (Q_1 Q) H (Z_1 Z)^T,$$
$$Q_1 B Z_1^T = (Q_1 Q) T (Z_1 Z)^T.$$

4   References

Golub G H and van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Moler C B and Stewart G W (1973) An algorithm for generalized matrix eigenproblems *SIAM J. Numer. Anal.* **10** 241–256

5 Parameters

1: COMPQ – CHARACTER*1 *Input*

On entry: specifies the form of the computed orthogonal matrix Q, as follows:

      if COMPQ = 'N', do not compute Q;

      if COMPQ = 'I', the orthogonal matrix Q is returned;

      if COMPQ = 'V', Q must contain an orthogonal matrix Q_1, and the product Q_1Q is returned.

Constraint: COMPQ = 'N', 'I' or 'V'.

2: COMPZ – CHARACTER*1 *Input*

On entry: specifies the form of the computed orthogonal matrix Z, as follows:

      if COMPZ = 'N', do not compute Z;

      if COMPZ = 'I', the orthogonal matrix Z is returned;

      if COMPZ = 'V', Z must contain an orthogonal matrix Z_1, and the product Z_1Z is returned.

Constraint: COMPZ = 'N', 'I' or 'V'.

3: N – INTEGER *Input*

On entry: n, the order of the matrices A and B.

Constraint: N ≥ 0.

4: ILO – INTEGER *Input*
5: IHI – INTEGER *Input*

On entry: i_{lo} and i_{hi} as determined by a previous call to F08WHF (SGGBAL/DGGBAL). Otherwise, they should be set to 1 and n, respectively.

Constraints:

      $1 \leq$ ILO $\leq$ IHI $\leq$ N if N > 0,
      ILO = 1 and IHI = 0 if N = 0.

6: A(LDA,*) – *real* array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, N)$.

On entry: the matrix A of the matrix pair (A, B). Usually, this is the matrix A returned by F08AGF (SORMQR/DORMQR).

On exit: A is overwritten by the upper Hessenberg matrix H.

7: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F08WEF (SGGHRD/DGGHRD) is called.

Constraint: LDA $\geq \max(1, N)$.

8: B(LDB,*) – *real* array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, N)$.

On entry: the upper triangular matrix B of the matrix pair (A, B). Usually, this is the matrix B returned by the QR factorization routine F08AEF (SGEQRF/DGEQRF).

On exit: the array B is overwritten by the upper triangular matrix T.

9:     LDB – INTEGER   *Input*

   On entry: the first dimension of the array B as declared in the (sub)program from which F08WEF (SGGHRD/DGGHRD) is called.

   Constraint: $\text{LDB} \geq \max(1, \text{N})$.

10:    Q(LDQ,*) – **real** array   *Input/Output*

   Note: the second dimension of the array Q must be at least $\max(1, \text{N})$.

   On entry: if COMPQ = 'N', Q is not referenced; if COMPQ = 'V', Q must contain an orthogonal matrix Q_1.

   On exit: if COMPQ = 'I', Q contains the orthogonal matrix Q; if COMPQ = 'V', Q is overwritten by $Q_1 Q$.

11:    LDQ – INTEGER   *Input*

   On entry: the first dimension of the array Q as declared in the (sub)program from which F08WEF (SGGHRD/DGGHRD) is called.

   Constraints:

   $\text{LDQ} \geq 1$ if COMPQ = 'N',
   $\text{LDQ} \geq \max(1, \text{N})$ if COMPQ = 'I' or 'V'.

12:    Z(LDZ,*) – **real** array   *Input/Output*

   Note: the second dimension of the array Z must be at least $\max(1, \text{N})$.

   On entry: if COMPZ = 'N', Z is not referenced; if COMPZ = 'V', Z must contain an orthogonal matrix Z_1.

   On exit: if COMPZ = 'I', Z contains the orthogonal matrix Z; if COMPZ = 'V', Z is overwritten by $Z_1 Z$.

13:    LDZ – INTEGER   *Input*

   On entry: the first dimension of the array Z as declared in the (sub)program from which F08WEF (SGGHRD/DGGHRD) is called.

   Constraints:

   $\text{LDZ} \geq 1$ if COMPZ = 'N',
   $\text{LDZ} \geq \max(1, \text{N})$ if COMPZ = 'V' or 'I'.

14:    INFO – INTEGER   *Output*

   On exit: INFO = 0 unless the routine detects an error (see Section 6).

6   Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

   If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7   Accuracy

The reduction to the generalized Hessenberg form is implemented using orthogonal transformations which are backward stable.

8 Further Comments

This routine is usually followed by F08XEF (SHGEQZ/DHGEQZ) which implements the QZ algorithm for computing generalized eigenvalues of a reduced pair of matrices.

The complex analogue of this routine is F08WSF (CGGHRD/ZGGHRD).

9 Example

See Section 9 of the documents for F08XEF (SHGEQZ/DHGEQZ) and F08YKF (STGEVC/DTGEVC).