G01FDF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

G01FDF returns the deviate associated with the given lower tail probability of the F or variance-ratio distribution with real degrees of freedom, via the routine name.

2 Specification

```
real FUNCTION G01FDF(P, DF1, DF2, IFAIL)
INTEGER              IFAIL
real                 P, DF1, DF2
```

3 Description

The deviate, f_p, associated with the lower tail probability, p, of the F-distribution with degrees of freedom ν_1 and ν_2 is defined as the solution to:

$$P(F \leq f_p : \nu_1, \nu_2) = p$$

$$= \frac{\nu_1^{\nu_1/2}\nu_1\nu_2^{\nu_2/2}\nu_2\Gamma((\nu_1+\nu_2)/2)}{\Gamma(\nu_1/2)\Gamma(\nu_2/2)} \int_0^{f_p} F^{(\nu_1-2)/2}(\nu_2+\nu_1 F)^{(\nu_1+\nu_2)/2} dF,$$

where $\nu_1, \nu_2 > 0$; $0 \leq f_p < \infty$.

The value of f_p is computed by means of a transformation to a beta distribution, $P_\beta(B \leq \beta : a, b)$:

$$P(F \leq f : \nu_1, \nu_2) = P_\beta\left(B \leq \frac{\nu_1 f}{\nu_1 f + \nu_2} : \nu_1/2, \nu_2/2\right)$$

and using a call to G01FEF.

For very large values of both ν_1 and ν_2, greater than 10^5, a normal approximation is used. If only one of ν_1 or ν_2 is greater than 10^5 then a χ^2 approximation is used, see Abramowitz and Stegun [1].

4 References

[1] Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

[2] Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworths

5 Parameters

1: P — ***real*** *Input*

On entry: the probability, p, from the required F-distribution.

Constraint: $0.0 \leq \mathrm{P} < 1.0$.

2: DF1 — ***real*** *Input*

On entry: the degrees of freedom of the numerator variance, ν_1.

Constraint: $\mathrm{DF1} > 0.0$.

3: DF2 — ***real*** *Input*

On entry: the degrees of freedom of the denominator variance, ν_2.

Constraint: $\mathrm{DF2} > 0.0$.

4: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, −1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error or gives a warning (see Section 6).

For this routine, because the values of output parameters may be useful even if IFAIL ≠ 0 on exit, users are recommended to set IFAIL to −1 before entry. **It is then essential to test the value of IFAIL on exit**.

6 Error Indicators and Warnings

If on entry IFAIL = 0 or −1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings specified by the routine:

If on exit IFAIL = 1, 2 or 4, then G01FDF returns 0.0.

IFAIL = 1

On entry, P < 0.0,
or P ≥ 1.0.

IFAIL = 2

On entry, DF1 ≤ 0.0,
or DF2 ≤ 0.0.

IFAIL = 3

The solution has not converged. The result should still be a reasonable approximation to the solution. Alternatively, G01FEF can be used with a suitable setting of the parameter TOL.

IFAIL = 4

The value of P is too close to 0 or 1 for the value of f_p to be computed. This will only occur when the large sample approximations are used.

7 Accuracy

The result should be accurate to 5 significant digits.

8 Further Comments

For higher accuracy G01FEF can be used along with the transformations given in Section 3.

9 Example

Lower tail probabilities are read for several F-distributions, and the corresponding deviates calculated and printed, until the end of data is reached.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G01FDF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             DF1, DF2, F, P
      INTEGER          IFAIL
*     .. External Functions ..
      real             G01FDF
      EXTERNAL         G01FDF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01FDF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     P      DF1     DF2      F'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) P, DF1, DF2
      IFAIL = -1
*
      F = G01FDF(P,DF1,DF2,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
         WRITE (NOUT,99999) P, DF1, DF2, F
      ELSE
         WRITE (NOUT,99999) P, DF1, DF2, F, ' NOTE: IFAIL = ', IFAIL
      END IF
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,4F8.3,A,I1)
      END
```

9.2 Program Data

```
G01FDF Example Program Data
0.9837  10.0   25.5           :P DF1 DF2
0.9000  1.0    1.0            :P DF1 DF2
0.5342  20.25  1.0            :P DF1 DF2
```

9.3 Program Results

```
 G01FDF Example Program Results

      P      DF1     DF2      F

    0.984  10.000  25.500   2.837
    0.900   1.000   1.000  39.866
    0.534  20.250   1.000   2.500
```