G04DBF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

G04DBF computes simultaneous confidence intervals for the differences between means. It is intended for use after G04BBF or G04BCF.

2 Specification

```
      SUBROUTINE G04DBF(TYPE, NT, TMEAN, RDF, C, LDC, CLEVEL, CIL, CIU,
     1                  ISIG, IFAIL)
      INTEGER           NT, LDC, ISIG(NT*(NT-1)/2), IFAIL
      real              TMEAN(NT), RDF, C(LDC,NT), CLEVEL,
     1                  CIL(NT*(NT-1)/2), CIU(NT*(NT-1)/2)
      CHARACTER*1       TYPE
```

3 Description

In the computation of analysis of a designed experiment the first stage is to compute the basic analysis of variance table, the estimate of the error variance (the residual or error mean square), $\hat{\sigma}^2$, the residual degress of freedom, ν, and the (variance ratio) F-statistic for the t treatments. The second stage of the analysis is to compare the treatment means. If the treatments have no structure, for example the treatments are different varieties, rather than being structured, for example a set of different temperatures, then a multiple comparison procedure can be used.

A multiple comparison procedure looks at all possible pairs of means and either computes confidence intervals for the difference in means or performs a suitable test on the difference. If there are t treatments then there are $t(t-1)/2$ comparisons to be considered. In tests the type 1 error or significance level is the probability that the result is considered to be significant when there is no difference in the means. If the usual t-test is used with, say, a five percent significance level then the type 1 error for all $k = t(t-1)/2$ tests will be much higher. If the tests were independent then if each test is carried out at the 100α percent level then the overall type 1 error would be $\alpha^* = 1-(1-\alpha)^k \simeq k\alpha$. In order to provide an overall protection the individual tests, or confidence intervals, would have to be carried out at a value of α such that α^* is the required significance level, e.g., five percent.

The $100(1-\alpha)$ percent confidence interval for the difference in two treatment means, $\hat{\tau}_i$ and $\hat{\tau}_j$ is given by

$$(\hat{\tau}_i - \hat{\tau}_j) \pm T^*_{(\alpha,\nu,t)} se(\hat{\tau}_i - \hat{\tau}_j),$$

where $se()$ denotes the standard error of the difference in means and $T^*_{(\alpha,\nu,t)}$ is an appropriate percentage point from a distribution. There are several possible choices for $T^*_{(\alpha,\nu,t)}$. These are:

(a) $\frac{1}{2}q_{(1-\alpha,\nu,t)}$, the studentised range statistic, see G01FMF. It is the appropriate statistic to compare the largest mean with the smallest mean. This is known as Tukey–Kramer method.

(b) $t_{(\alpha/k,\nu)}$, this is the Bonferroni method.

(c) $t_{(\alpha_0,\nu)}$, where $\alpha_0 = 1-(1-\alpha)^{1/k}$, this is known as the Dunn–Sidak method.

(d) $t_{(\alpha,\nu)}$, this is known as Fisher's LSD (least significant difference) method. It should only be used if the overall F-test is significant, the number of treatment comparisons is small and were planned before the analysis.

(e) $\sqrt{(k-1)F_{1-\alpha,k-1,\nu}}$ where $F_{1-\alpha,k-1,\nu}$ is the deviate corresponding to a lower tail probability of $1-\alpha$ from an F-distribution with $k-1$ and ν degrees of freedom. This is Scheffe's method.

In cases (b), (c) and (d), $t_{(\alpha,\nu)}$ denotes the α two-tail significance level for the Student's t-distribution with ν degrees of freedom, see G01FBF.

The Scheffe method is the most conservative, followed closely by the Dunn–Sidak and Tukey–Kramer methods.

To compute a test for the difference between two means the statistic,

$$\frac{\hat{\tau}_i - \hat{\tau}_j}{se(\hat{\tau}_i - \hat{\tau}_j)}$$

is compared with the appropriate value of $T^*_{(\alpha,\nu,t)}$.

4 References

[1] Kotz S and Johnson N L (ed.) (1985) Multiple range and associated test procedures *Encyclopedia of Statistical Sciences* **5** Wiley, New York

[2] Kotz S and Johnson N L (ed.) (1985) Multiple comparison *Encyclopedia of Statistical Sciences* **5** Wiley, New York

[3] Winer B J (1970) *Statistical Principles in Experimental Design* McGraw–Hill

5 Parameters

1: TYPE — CHARACTER*1 *Input*

On entry: indicates which method is to be used.

If TYPE = 'T', the Tukey–Kramer method is used.
If TYPE = 'B', the Bonferroni method is used.
If TYPE = 'D', the Dunn–Sidak method is used.
If TYPE = 'L', the Fisher LSD method is used.
If TYPE = 'S', the Scheffe's method is used.

Constraint: TYPE = 'T','B','D', 'L' or 'S'.

2: NT — INTEGER *Input*

On entry: the number of treatment means, t.

Constraint: NT $\geq$ 2.

3: TMEAN(NT) — ***real*** array *Input*

On entry: the treatment means, $\hat{\tau}_i$, $i = 1, 2, \ldots, t$.

4: RDF — ***real*** *Input*

On entry: the residual degrees of freedom, ν.

Constraint: RDF $\geq$ 1.0.

5: C(LDC,NT) — ***real*** array *Input*

On entry: the strictly lower triangular part of C must contain the standard errors of the differences between the means as returned by G04BBF and G04BCF. That is C(i,j), $i > j$, contains the standard error of the difference between the ith and jth mean in TMEAN.

Constraint: C$(i,j) > 0.0$, $i = 2, 3, \ldots, t$; $j = 1, 2, \ldots, i-1$.

6: LDC — INTEGER *Input*

On entry: the first dimension of the array C as declared in the (sub)program from which G04DBF is called.

Constraint: LDC $\geq$ NT.

7: CLEVEL — ***real*** *Input*

On entry: the required confidence level for the computed intervals, $(1-\alpha)$.

Constraint: 0.0 < CLEVEL < 1.0.

8: CIL(NT*(NT−1)/2) — ***real*** array *Output*

On exit: the $((i-1)(i-2)/2+j)$th element contains the lower limit to the confidence interval for the difference between ith and jth means in TMEAN, $i = 2, 3, \ldots, t$; $j = 1, 2, \ldots, i-1$.

9: CIU(NT*(NT−1)/2) — ***real*** array *Output*

On exit: the $((i-1)(i-2)/2+j)$th element contains the upper limit to the confidence interval for the difference between ith and jth means in TMEAN, $i = 2, 3, \ldots, t$; $j = 1, 2, \ldots, i-1$.

10: ISIG(NT*(NT−1)/2) — INTEGER array *Output*

On exit: the $((i-1)(i-2)/2+j)$th element indicates if the difference between ith and jth means in TMEAN is significant, $i = 2, 3, \ldots, t$; $j = 1, 2, \ldots, i-1$. If the difference is significant then the returned value is 1; otherwise the returned value is 0.

11: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, −1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Errors and Warnings

If on entry IFAIL = 0 or −1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings specified by the routine:

IFAIL = 1

On entry, NT < 2,
or LDC < NT,
or RDF < 1.0,
or CLEVEL $\leq$ 0.0,
or CLEVEL $\geq$ 1.0,
or TYPE $\neq$ 'T', 'B', 'D', 'L' or 'S'.

IFAIL = 2

On entry, C$(i,j) \leq 0.0$ for some i, j, $i = 2, 3, \ldots, t$; $j = 1, 2, \ldots, i-1$.

IFAIL = 3

There has been a failure in the computation of the studentized range statistic. This is an unlikely error. Try using a small value of CLEVEL.

7 Accuracy

For the accuracy of the percentage point statistics see G01FMF and G01FBF.

8 Further Comments

If the treatments have a structure then the use of linear contrasts as computed by G04DAF may be more appropriate.

An alternative approach to one used in this routine is the sequential testing of the Student–Newman–Keuls procedure. This, in effect, uses the Tukey–Kramer method but first ordering the treatment means and examining only subsets of the treatment means in which the largest and smallest are significantly different. At each stage the third parameter of the Studentised range statistic is the number of means in the subset rather than the total number of means.

9 Example

In the example taken from Winer [3] a completely randomised design with unequal treatment replication is analysed using G04BBF and then confidence intervals are computed by G04DBF using the Tukey–Kramer method.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G04DBF Example Program Text
*     Mark 17 Release. NAG Copyright 1995.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, NTMAX, NBMAX
      PARAMETER        (NMAX=26,NTMAX=4,NBMAX=1)
*     .. Local Scalars ..
      real             CLEVEL, GMEAN, RDF, TOL
      INTEGER          I, IFAIL, IJ, IRDF, J, N, NBLOCK, NT
      CHARACTER        TYPE
*     .. Local Arrays ..
      real             BMEAN(NBMAX), C(NTMAX,NTMAX),
     +                 CIL(NTMAX*(NTMAX-1)/2), CIU(NTMAX*(NTMAX-1)/2),
     +                 EF(NTMAX), R(NMAX), TABLE(4,5), TMEAN(NTMAX),
     +                 WK(NTMAX*NTMAX+NTMAX), Y(NMAX)
      INTEGER          IREP(NTMAX), ISIG(NTMAX*(NTMAX-1)/2), IT(NMAX)
      CHARACTER        STAR(2)
*     .. External Subroutines ..
      EXTERNAL         G04BBF, G04DBF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G04DBF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, NT
      IF (N.LE.NMAX .AND. NT.LE.NTMAX) THEN
         READ (NIN,*) (Y(I),I=1,N)
         READ (NIN,*) (IT(I),I=1,N)
         TOL = 0.000005e0
         IRDF = 0
         NBLOCK = 1
         IFAIL = -1
         CALL G04BBF(N,Y,NBLOCK,NT,IT,GMEAN,BMEAN,TMEAN,TABLE,4,C,NTMAX,
     +               IREP,R,EF,TOL,IRDF,WK,IFAIL)
         WRITE (NOUT,*)
         WRITE (NOUT,*) ' ANOVA table'
         WRITE (NOUT,*)
         WRITE (NOUT,*)
     +     '  Source        df         SS           MS           F',
     +     '         Prob'
         WRITE (NOUT,*)
         WRITE (NOUT,99998) ' Treatments', (TABLE(2,J),J=1,5)
         WRITE (NOUT,99998) ' Residual  ', (TABLE(3,J),J=1,3)
         WRITE (NOUT,99998) ' Total     ', (TABLE(4,J),J=1,2)
         WRITE (NOUT,*)
         WRITE (NOUT,*) ' Treatment means'
         WRITE (NOUT,*)
```

```
         WRITE (NOUT,99999) (TMEAN(J),J=1,NT)
         WRITE (NOUT,*)
         WRITE (NOUT,*) ' Simultaneous Confidence Intervals'
         WRITE (NOUT,*)
         RDF = TABLE(3,1)
         READ (NIN,*) TYPE, CLEVEL
*
         CALL G04DBF(TYPE,NT,TMEAN,RDF,C,NTMAX,CLEVEL,CIL,CIU,ISIG,
     +               IFAIL)
*
         STAR(2) = '*'
         STAR(1) = ' '
         IJ = 0
         DO 40 I = 1, NT
            DO 20 J = 1, I - 1
               IJ = IJ + 1
               WRITE (NOUT,99997) I, J, CIL(IJ), CIU(IJ),
     +           STAR(ISIG(IJ)+1)
   20       CONTINUE
   40    CONTINUE
      END IF
      STOP
*
99999 FORMAT (10F8.3)
99998 FORMAT (A,3X,F3.0,2X,2(F10.1,2X),F10.3,2X,F9.4)
99997 FORMAT (2X,2I2,3X,2(F10.3,3X),A)
      END
```

9.2 Program Data

```
G04DBF Example Program Data

26 4

 3  2  4  3  1  5
 7  8  4 10  6
 3  2  1  2  4  2  3  1
10 12  8  5 12 10  9

1 1 1 1 1 1
2 2 2 2 2
3 3 3 3 3 3 3 3
4 4 4 4 4 4 4

'T' .95
```

9.3 Program Results

```
G04DBF Example Program Results

 ANOVA table

  Source        df          SS          MS          F         Prob

Treatments    3.       239.9        80.0      24.029     0.0000
Residual     22.        73.2         3.3
Total        25.       313.1

 Treatment means

  3.000   7.000   2.250   9.429

 Simultaneous Confidence Intervals

  2 1        0.933        7.067   *
  3 1       -3.486        1.986
  3 2       -7.638       -1.862   *
  4 1        3.610        9.247   *
  4 2       -0.538        5.395
  4 3        4.557        9.800   *
```