NAG Fortran Library Routine Document

G05HNF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1 Purpose

G05HNF generates a given number of terms of an exponential GARCH(p, q) process (see Engle and Ng (1993)).

2 Specification

```
SUBROUTINE G05HNF(DIST, NUM, IP, IQ, THETA, DF, HT, ET, FCALL, RVEC,
1                  IGEN, ISEED, RWSAV, IFAIL)
INTEGER           NUM, IP, IQ, IGEN, ISEED(4), IFAIL
real              THETA(2*IQ+IP+1), DF, HT(NUM), ET(NUM), RVEC(40),
1                  RWSAV(9)
LOGICAL           FCALL
CHARACTER*1       DIST
```

3 Description

An exponential GARCH(p, q) process is represented by:

$$ln(h_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i z_{t-i} + \sum_{i=1}^{q} \phi_i(|z_{t-i}| - E[|z_{t-i}|]) + \sum_{j=1}^{p} \beta_j ln(h_{t-j}), \quad t = 1, \dots, T,$$

where $z_t = \dfrac{\epsilon_t}{\sqrt{h_t}}$, $E[|z_{t-i}|]$ denotes the expected value of $|z_{t-i}|$, and $\epsilon_t|\psi_{t-1} = N(0, h_t)$ or $\epsilon_t|\psi_{t-1} = S_t(df, h_t)$. Here S_t is a standardised Student's t-distribution with df degrees of freedom and variance h_t, T is the number of observations in the sequence, ϵ_t is the *observed* value of the GARCH(p, q) process at time t, h_t is the conditional variance at time t, and ψ_t the set of all information up to time t.

One of the initialisation routines G05KBF (for a repeatable sequence if computed sequentially) or G05KCF (for a non-repeatable sequence) must be called prior to the first call to G05HNF.

4 References

Engle R (1982) Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation *Econometrica* **50** 987–1008

Bollerslev T (1986) Generalised autoregressive conditional heteroskedasticity *Journal of Econometrics* **31** 307–327

Engle R and Ng V (1993) Measuring and Testing the Impact of News on Volatility *Journal of Finance* **48** 1749–1777

Hamilton J (1994) *Time Series Analysis* Princeton University Press

Glosten L, Jagannathan R and Runkle D (1993) Relationship between the expected value and the volatility of nominal excess return on stocks *Journal of Finance* **48** 1779–1801

5   Parameters

1:    DIST – CHARACTER*1   *Input*

On entry: the type of distribution to use for ϵ_t.

DIST = 'N'

Then a Normal distribution is used.

DIST = 'T'

Then a Student's t-distribution is used.

Constraint: DIST = 'N' or 'T'.

2:    NUM – INTEGER   *Input*

On entry: the number of terms in the sequence, T.

Constraint: NUM > 0.

3:    IP – INTEGER   *Input*

On entry: the number of coefficients, β_i, for $i = 1, \ldots, p$.

Constraints:

$2 \times \text{IQ} + \text{IP} + 1 \le 20$,
$\text{IP} \ge 0$.

4:    IQ – INTEGER   *Input*

On entry: the number of coefficients, α_i, for $i = 1, \ldots, q$.

Constraints:

$2 \times \text{IQ} + \text{IP} + 1 \le 20$,
$\text{IQ} \ge 1$.

5:    THETA(2*IQ+IP+1) – ***real*** array   *Input*

On entry: the initial parameter estimates for the vector θ. The first element must contain the coefficient α_o and the next IQ elements must contain the autoregressive coefficients α_i, for $i = 1, \ldots, q$. The next IQ elements must contain the coefficients ϕ_i, for $i = 1, \ldots, q$. The next IP elements must contain the moving average coefficients β_j, for $j = 1, \ldots, p$.

6:    DF – ***real***   *Input*

On entry: the number of degrees of freedom for the Student's t-distribution. It is not referenced if DIST = 'N'.

Constraint: DF > 2.

7:    HT(NUM) – ***real*** array   *Output*

On exit: the conditional variances h_t, for $t = 1, \ldots, T$ for the GARCH(p, q) sequence.

8:    ET(NUM) – ***real*** array   *Output*

On exit: the observations ϵ_t, for $t = 1, \ldots, T$ for the GARCH(p, q) sequence.

9:    FCALL – LOGICAL   *Input*

On entry: if FCALL = .TRUE., a new sequence is to be generated, otherwise a given sequence is to be continued using the information in RVEC.

10:   RVEC(40) – **real** array *Input/Output*

On entry: the array contains information required to continue a sequence if FCALL = .FALSE..

On exit: contains information that can be used in a subsequent call of G05HNF, with FCALL = .FALSE..

11:   IGEN – INTEGER *Input*

On entry: must contain the identification number for the generator to be used to return a pseudo-random number and should remain unchanged following initialisation by a prior call to one of the routines G05KBF or G05KCF.

12:   ISEED(4) – INTEGER array *Input/Output*

On entry: contains values which define the current state of the selected generator.

On exit: contains updated values defining the new state of the selected generator.

13:   RWSAV(9) – **real** array *Workspace*

14:   IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, −1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value −1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL $\neq$ 0 on exit, the recommended value is −1. **When the value −1 or 1 is used it is essential to test the value of IFAIL on exit.**

6   Error Indicators and Warnings

If on entry IFAIL = 0 or −1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, IP < 0,
or   IQ < 1,
or   DF ≤ 2,
or   NUM ≤ 0,
or   DIST $\neq$ 'N' and DIST $\neq$ 'T',
or $2 \times$ IQ $+$ IP $+ 1 > 20$.

IFAIL = 2

Invalid sequence generated, use different parameters.

7   Accuracy

Not applicable.

8   Further Comments

None.

9 Example

See Section 9 of the document for G13FGF.