G13BBF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

G13BBF filters a time series by a transfer function model.

2 Specification

```
SUBROUTINE G13BBF(Y, NY, MR, NMR, PAR, NPAR, CY, WA, IWA, B, NB,
1                 IFAIL)
INTEGER           NY, MR(NMR), NMR, NPAR, IWA, NB, IFAIL
real              Y(NY), PAR(NPAR), CY, WA(IWA), B(NB)
```

3 Description

From a given series $y_1, y_2, \ldots, y_n$ a new series $b_1, b_2, \ldots, b_n$ is calculated using a supplied (filtering) transfer function model according to the equation

$$b_t = \delta_1 b_{t-1} + \delta_2 b_{t-2} + \ldots + \delta_p b_{t-p} + \omega_0 y_{t-b} - \omega_1 y_{t-b-1} - \ldots - \omega_q y_{t-b-q} \tag{1}$$

As in the use of G13BAF large transient errors may arise in the early values of b_t due to ignorance of y_t for $t < 0$, and two possibilities are allowed:

(i) The equation (1) is applied from $t = 1 + b + q, \ldots, n$ so all terms in y_t on the right-hand side of (1) are known, the unknown set of values b_t for $t = b + q, \ldots, b + q + 1 - p$ being taken as zero.
(ii) The unknown values of y_t for $t \leq 0$ are estimated by backforecasting exactly as for G13BAF.

4 References

[1] Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* Holden–Day (Revised Edition)

5 Parameters

1: Y(NY) — ***real*** array *Input*

On entry: the Q'_y backforecasts starting with backforecast at time $1 - Q'_y$ to backforecast at time 0 followed by the time series starting at time 1, where $Q'_y = \mathrm{MR}(6) + \mathrm{MR}(9) \times \mathrm{MR}(10)$. If there are no backforecasts either because the ARIMA model for the time series is not known or because it is known but has no moving average terms, then the time series starts at the beginning of Y.

2: NY — INTEGER *Input*

On entry: the total number of backforecasts and time series data points in array Y.

Constraint: $\mathrm{NY} \geq \max(1 + Q'_y, \mathrm{NPAR})$.

3: MR(NMR) — INTEGER array *Input*

On entry: the orders vector for the filtering TF model followed by the orders vector for the ARIMA model for the time series if the latter is known. The TF model orders appear in the standard form (b, q, p) as given in the Chapter Introduction. Note that if the ARIMA model for the time series is supplied then the routine will assume that the first Q'_y values of the array Y are backforecasts.

Constraints: the filtering model is restricted in the following way:

$\mathrm{MR}(1), \mathrm{MR}(2), \mathrm{MR}(3) \geq 0$.

the ARIMA model for the time series is restricted in the following ways:

MR(k) ≥ 0; $k = 4, \ldots, 10$;
MR(7) + MR(8) + MR(9) = 0 if and only if MR(10) = 0;

MR(10) $\neq 1$.

4: NMR — INTEGER *Input*

On entry: the number of values supplied in the array MR. It takes the value 3 if no ARIMA model for the time series is supplied but otherwise it takes the value 10. Thus NMR acts as an indicator as to whether backforecasting can be carried out.

Constraint: NMR = 3 or 10.

5: PAR(NPAR) — ***real*** array *Input*

On entry: the parameters of the filtering TF model followed by the parameters of the ARIMA model for the time series. In the TF model the parameters are in the standard order of MA-like followed by AR-like operator parameters. In the ARIMA model the parameters are in the standard order of non-seasonal AR and MA followed by seasonal AR and MA.

6: NPAR — INTEGER *Input*

On entry: the total number of parameters held in array PAR.

Constraints:

if NMR = 3, NPAR = MR(2) + MR(3) + 1,
if NMR = 10, NPAR = MR(2) + MR(3) + 1 + MR(4) + MR(6) + MR(7) + MR(9).

7: CY — ***real*** *Input*

On entry: if the ARIMA model is known (i.e., NMR = 10), CY must specify the constant term of the ARIMA model for the time series. If this model is not known (i.e., NMR = 3) then CY is not used.

8: WA(IWA) — ***real*** array *Workspace*

9: IWA — INTEGER *Input*

On entry: the dimension of the array WA as declared in the (sub)program from which G13BBF is called.

Constraints:

let K = MR(3) + MR(4) + MR(5) + (MR(7)} + MR(8))} × MR(10)} then,
if NMR = 3, IWA $\geq$ MR(1) + NPAR,
if NMR = 10, IWA $\geq$ MR(1) + NPAR + K × (K+2).

10: B(NB) — ***real*** array *Output*

On exit: the filtered output series. If the ARIMA model for the time series was known and hence Q'_y backforecasts were supplied in Y then B contains Q'_y 'filtered' backforecasts followed by the filtered series. Otherwise the filtered series begins at the start of B just as the original series began at the start of Y. In either case if the value of the series at time t is held in Y(t) then the filtered value at time t is held in B(t).

11: NB — INTEGER *Input*

On entry: the dimension of the array B as declared in the (sub)program from which G13BBF is called.

In addition to holding the returned filtered series, B is also used as an intermediate work array if the ARIMA model for the time series is known.

Constraints:

if NMR = 3, NB $\geq$ NY,
if NMR = 10, NB $\geq$ NY + max(MR(1) + MR(2), MR(3)).

12: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, −1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors detected by the routine:

IFAIL = 1

On entry, NMR $\neq 3$ and NMR $\neq 10$,
or MR(i) < 0, $i = 1, 2, \ldots$,NMR,
or NMR = 10 and MR(10) = 1,
or NMR = 10 and MR(10) = 0 and MR(7) + MR(8) + MR(9) $\neq 0$,
or NMR = 10 and MR(10) $\neq 0$, and MR(7) + MR(8) + MR(9) = 0,
or NPAR is inconsistent with the contents of MR,
or WA is too small,
or B is too small.

IFAIL = 2

A supplied model has parameter values which have failed the validity test.

IFAIL = 3

The supplied time series is too short to carry out the requested filtering successfully.

IFAIL = 4

This only occurs when an ARIMA model for the time series has been supplied. The matrix which is used to solve for the starting values for MA filtering is singular.

7 Accuracy

Accuracy and stability are high except when the AR-like parameters are close to the invertibility boundary. All calculations are performed in ***basic precision*** except for one inner product type calculation which on machines of low precision is performed in ***additional precision***.

8 Further Comments

The time taken by the routine is roughly proportional to the product of the length of the series and number of parameters in the filtering model with appreciable increase if an ARIMA model is supplied for the time series.

9 Example

The example program reads a time series of length 296. It reads one univariate ARIMA (1,1,0,0,1,1,12) model for the series and the (0,13,12) transfer function filtering model. 12 initial backforecasts are required and these are calculated by a call to G13AJF. The backforecasts are inserted at the start of the series and G13BBF is called to perform the filtering.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G13BBF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NXMAX, NPMAX, ISTMAX, IFVMAX, IW, IQXDAX, NYMAX,
     +                 NBMAX
      PARAMETER        (NXMAX=200,NPMAX=30,ISTMAX=30,IFVMAX=12,IW=2000,
     +                 IQXDAX=15,NYMAX=NXMAX+IQXDAX,NBMAX=NYMAX+20)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             A1, A2, CX, CY, RMS
      INTEGER          I, IDD, IFAIL, II, IJ, IQXD, IWA, J, K, N, NB,
     +                 NI, NMR, NPAR, NPARX, NST, NX, NY
*     .. Local Arrays ..
      real             B(NXMAX), FSD(IFVMAX), FVA(IFVMAX), PAR(NPMAX),
     +                 PARX(NPMAX), ST(NPMAX), WA(IW), X(NXMAX),
     +                 Y(NYMAX)
      INTEGER          ISF(4), MR(10), MRX(7)
*     .. External Subroutines ..
      EXTERNAL         G13AJF, G13BBF
*     .. Intrinsic Functions ..
      INTRINSIC        MAX, MIN, MOD
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G13BBF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) NX
      WRITE (NOUT,*)
      IF (NX.GT.0 .AND. NX.LE.NXMAX) THEN
         READ (NIN,*) (X(I),I=1,NX)
*        Read univariate ARIMA for series
         READ (NIN,*) (MRX(I),I=1,7)
         READ (NIN,*) CX
         NPARX = MRX(1) + MRX(3) + MRX(4) + MRX(6)
         IF (NPARX.GT.0 .AND. NPARX.LE.NPMAX) THEN
            READ (NIN,*) (PARX(I),I=1,NPARX)
*           Read model by which to filter series
            READ (NIN,*) (MR(I),I=1,3)
            NPAR = MR(2) + MR(3) + 1
            IF (NPAR.GT.0 .AND. NPAR+NPARX.LE.NPMAX) THEN
               READ (NIN,*) (PAR(I),I=1,NPAR)
*              Initially backforecast QY values
*              (1) Reverse series in situ
               N = NX/2
               NI = NX
               DO 20 I = 1, N
                  A1 = X(I)
                  A2 = X(NI)
                  X(I) = A2
                  X(NI) = A1
                  NI = NI - 1
   20          CONTINUE
               IDD = MRX(2) + MRX(5)
*              (2) Possible sign reversal for ARIMA constant
```

```
               IF (MOD(IDD,2).NE.0) CX = -CX
*              (3) Calculate number of backforecasts required
               IQXD = MRX(3) + MRX(6)*MRX(7)
               IFAIL = 0
               IF (IQXD.NE.0) CALL G13AJF(MRX,PARX,NPARX,CX,0,X,NX,RMS,
     +                                    ST,ISTMAX,NST,IQXD,FVA,FSD,
     +                                    IFVMAX,ISF,WA,IW,IFAIL)
*              Move backforecasts to start of Y array
               J = IQXD
               DO 40 I = 1, IQXD
                  Y(I) = FVA(J)
                  J = J - 1
   40          CONTINUE
*              Move series into Y
               J = IQXD + 1
               K = NX
               DO 60 I = 1, NX
                  IF (J.GT.NYMAX) STOP
                  Y(J) = X(K)
                  J = J + 1
                  K = K - 1
   60          CONTINUE
            END IF
*           Calculate series length
            NY = NX + IQXD
*           Move ARIMA for series into MR
            DO 80 I = 1, 7
               MR(3+I) = MRX(I)
   80       CONTINUE
*           Move parameters of ARIMA for Y into PAR
            DO 100 I = 1, NPARX
               PAR(NPAR+I) = PARX(I)
  100       CONTINUE
            NPAR = NPAR + NPARX
*           Move constant and reset sign reversal
            CY = CX
            IF (MOD(IDD,2).NE.0) CY = -CY
*           Set parameters for call to filter routine G13BBF
            NMR = 10
            IWA = MR(3) + MR(4) + MR(5) + (MR(7)+MR(8))*MR(10)
            IWA = NPAR + IWA*(IWA+2)
            NB = NY + MAX(MR(1)+MR(2),MR(3))
            IF (IWA.LE.IW .AND. NB.LE.NBMAX) THEN
               IFAIL = 0
*
*              Filter series by call to G13BBF
               CALL G13BBF(Y,NY,MR,NMR,PAR,NPAR,CY,WA,IWA,B,NB,IFAIL)
*
               WRITE (NOUT,*)
     +           '                   Original        Filtered'
               WRITE (NOUT,*)
     +           ' Backforecasts    y-series         series'
               IF (IQXD.NE.0) THEN
                  IJ = -IQXD
                  DO 120 I = 1, IQXD
                     WRITE (NOUT,99999) IJ, Y(I), B(I)
                     IJ = IJ + 1
  120             CONTINUE
```

```
                  WRITE (NOUT,*)
                  WRITE (NOUT,*)
     +'        Filtered        Filtered        Filtered        Filtered'
                  WRITE (NOUT,*)
     + '         series          series          series          series'
                  DO 140 I = IQXD + 1, NY, 4
                     WRITE (NOUT,99998) (II-IQXD,B(II),II=I,MIN(NY,I+3))
  140             CONTINUE
                END IF
             END IF
          END IF
       END IF
      END IF
      STOP
*
99999 FORMAT (1X,I8,F17.1,F16.1)
99998 FORMAT (1X,I5,F10.1,I6,F10.1,I6,F10.1,I6,F10.1)
      END
```

9.2 Program Data

```
G13BBF Example Program Data
   158
5312. 5402. 4960. 4717. 4383. 3828. 3665. 3718.
3744. 3994. 4150. 4064. 4324. 4256. 3986. 3670.
3292. 2952. 2765. 2813. 2850. 3085. 3256. 3213.
3514. 3386. 3205. 3124. 2804. 2536. 2445. 2649.
2761. 3183. 3456. 3529. 4067. 4079. 4082. 4029.
3887. 3684. 3707. 3923. 4068. 4557. 4975. 5197.
6054. 6471. 6277. 5529. 5059. 4539. 4236. 4305.
4299. 4478. 4561. 4470. 4712. 4512. 4129. 3942.
3572. 3149. 3026. 3141. 3145. 3322. 3384. 3373.
3630. 3555. 3413. 3127. 2966. 2685. 2642. 2789.
2867. 3032. 3125. 3176. 3359. 3265. 3053. 2915.
2690. 2518. 2523. 2737. 3074. 3671. 4355. 4648.
5232. 5349. 5228. 5172. 4932. 4637. 4642. 4930.
5033. 5223. 5482. 5560. 5960. 5929. 5697. 5583.
5316. 5039. 4972. 5169. 5138. 5316. 5409. 5375.
5803. 5736. 5643. 5416. 5059. 4810. 4937. 5166.
5187. 5348. 5483. 5626. 6077. 6033. 5996. 5860.
5499. 5210. 5421. 5609. 5586. 3663. 5829. 6005.
6693. 6792. 6966. 7227. 7089. 6823. 7286. 7621.
7758. 8000. 8393. 8592. 9186. 9175.
     1     1     0     0     1     1    12
  0.000
  0.620  0.820
     0    13    12
 1.0131  0.0806 -0.0150 -0.0150 -0.0150 -0.0150
-0.0150 -0.0150 -0.0150 -0.0150 -0.0150 -0.0150
 0.9981 -0.0956  0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
 0.0000  0.8200
```

9.3 Program Results

G13BBF Example Program Results

Backforecasts	Original y-series	Filtered series
-12	5159.0	4549.2
-11	5165.9	4550.9
-10	4947.5	4552.8
-9	4729.8	4554.9
-8	4424.5	4557.4
-7	4072.5	4560.7
-6	3995.5	4565.0
-5	4142.7	4571.1
-4	4219.7	4580.0
-3	4452.1	4593.5
-2	4758.0	4614.3
-1	4834.6	4647.1

	Filtered series		Filtered series		Filtered series		Filtered series
1	4699.2	2	4782.2	3	4552.8	4	4550.4
5	4525.7	6	4324.8	7	4256.9	8	4169.7
9	4127.9	10	4154.6	11	4011.3	12	3878.7
13	3705.1	14	3619.1	15	3603.1	16	3496.1
17	3422.6	18	3463.5	19	3349.8	20	3262.1
21	3225.9	22	3218.1	23	3103.6	24	3023.5
25	2905.9	26	2758.5	27	2828.2	28	2958.4
29	2926.2	30	3019.8	31	3010.7	32	3082.8
33	3111.7	34	3286.3	35	3279.3	36	3324.4
37	3461.7	38	3468.3	39	3709.0	40	3839.6
41	4004.4	42	4146.3	43	4265.3	44	4344.6
45	4419.8	46	4647.2	47	4802.6	48	4999.5
49	5446.0	50	5861.0	51	5855.9	52	5310.7
53	5202.5	54	5046.6	55	4857.1	56	4812.3
57	4740.7	58	4631.1	59	4447.5	60	4317.7
61	4079.8	62	3833.7	63	3667.7	64	3774.8
65	3709.9	66	3648.5	67	3645.3	68	3619.8
69	3549.4	70	3439.2	71	3250.3	72	3209.2
73	3005.2	74	2912.4	75	2994.1	76	2947.9
77	3103.7	78	3168.1	79	3226.0	80	3224.1
81	3233.0	82	3119.2	83	2992.5	84	3014.8
85	2763.7	86	2671.3	87	2664.9	88	2778.2
89	2823.8	90	2989.0	91	3072.2	92	3132.1
93	3394.6	94	3717.4	95	4180.5	96	4405.9
97	4605.2	98	4733.0	99	4830.9	100	5030.8
101	5079.0	102	5125.0	103	5236.7	104	5392.7
105	5396.7	106	5300.7	107	5312.1	108	5336.6
109	5347.9	110	5331.2	111	5322.0	112	5444.8
113	5468.7	114	5532.9	115	5555.9	116	5603.4
117	5483.2	118	5406.8	119	5250.5	120	5171.9
121	5217.4	122	5162.3	123	5296.1	124	5268.2
125	5204.9	126	5290.7	127	5500.0	128	5552.3
129	5503.3	130	5419.2	131	5335.6	132	5447.6
133	5495.1	134	5475.1	135	5643.8	136	5713.1
137	5655.1	138	5691.9	139	5958.4	140	5959.0
141	5884.8	142	3714.7	143	5877.8	144	5814.1
145	6095.6	146	6210.7	147	6560.5	148	7013.9

```
149    7174.8   150    7230.8   151    7726.7   152    7880.0
153    7997.4   154    8428.5   155    8264.1   156    8443.1
157    8615.4   158    8644.6
```