

G13CCF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

G13CCF calculates the smoothed sample cross spectrum of a bivariate time series using one of four lag windows – rectangular, Bartlett, Tukey or Parzen window.

2 Specification

```

SUBROUTINE G13CCF(NXY, MTXY, PXY, IW, MW, IS, IC, NC, CXY, CYX,
1              KC, L, NXYG, XG, YG, NG, IFAIL)
INTEGER       NXY, MTXY, IW, MW, IS, IC, NC, KC, L, NXYG, NG,
1              IFAIL
real        PXY, CXY(NC), CYX(NC), XG(NXYG), YG(NXYG)

```

3 Description

The smoothed sample cross spectrum is a complex valued function of frequency ω , $f_{xy}(\omega) = cf(\omega) + iqf(\omega)$, defined by its real part or co-spectrum

$$cf(\omega) = \frac{1}{2\pi} \sum_{k=-M+1}^{M-1} w_k C_{xy}(k+S) \cos(\omega k)$$

and imaginary part or quadrature spectrum

$$qf(\omega) = \frac{1}{2\pi} \sum_{k=-M+1}^{M-1} w_k C_{xy}(k+S) \sin(\omega k)$$

where $w_k = w_{-k}$, $k = 0, 1, \dots, M-1$ is the smoothing lag window as defined in the description of G13CAF. The alignment shift S is recommended to be chosen as the lag k at which the cross covariances $c_{xy}(k)$ peak, so as to minimize bias.

The results are calculated for frequency values

$$\omega_j = \frac{2\pi j}{L}, \quad j = 0, 1, \dots, [L/2]$$

where $[]$ denotes the integer part.

The cross covariances $c_{xy}(k)$ may be supplied by the user, or constructed from supplied series x_1, x_2, \dots, x_n ; y_1, y_2, \dots, y_n as

$$c_{xy}(k) = \frac{\sum_{t=1}^{n-k} x_t y_{t+k}}{n}, \quad k \geq 0$$

$$c_{xy}(k) = \frac{\sum_{t=1-k}^n x_t y_{t+k}}{n} = c_{yx}(-k), \quad k < 0$$

this convolution being carried out using the finite Fourier transform.

The supplied series may be mean and trend corrected and tapered before calculation of the cross covariances, in exactly the manner described in G13CAF for univariate spectrum estimation. The results are corrected for any bias due to tapering.

The bandwidth associated with the estimates is not returned. It will normally already have been calculated in previous calls of G13CAF for estimating the univariate spectra of y_t and x_t .

4 References

- [1] Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden-Day
 [2] Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

5 Parameters

- 1:** NXY — INTEGER *Input*
On entry: the length, n , of the time series x and y .
Constraint: $NXY \geq 1$.
- 2:** MTXY — INTEGER *Input*
On entry: if cross covariances are to be calculated by the routine ($IC = 0$), MTXY must specify whether the data is to be initially mean or trend corrected.
 MTXY = 0 for no correction,
 MTXY = 1 for mean correction,
 MTXY = 2 for trend correction,
 if cross covariances are supplied ($IC \neq 0$), MTXY is not used.
Constraint: $0 \leq MTXY \leq 2$ if $IC = 0$.
- 3:** PXY — *real* *Input*
On entry: if cross covariances are to be calculated by the routine ($IC = 0$), PXY must specify the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper. A value of 0.0 implies no tapering. If cross covariances are supplied ($IC \neq 0$), PXY is not used.
Constraint: $0.0 \leq PXY \leq 1.0$, if $IC = 0$.
- 4:** IW — INTEGER *Input*
On entry: the choice of lag window. $IW = 1$ for rectangular, 2 for Bartlett, 3 for Tukey or 4 for Parzen.
Constraint: $1 \leq IW \leq 4$.
- 5:** MW — INTEGER *Input*
On entry: the ‘cut-off’ point, M , of the lag window, relative to any alignment shift that has been applied. Windowed cross covariances at lags $(-MW + IS)$ or less, and at lags $(MW + IS)$ or greater are zero.
Constraints:
 $MW \geq 1$,
 $MW + |IS| \leq NXY$.
- 6:** IS — INTEGER *Input*
On entry: the alignment shift, S , between the x and y series. If x leads y , the shift is positive.
Constraint: $-MW < IS < MW$.
- 7:** IC — INTEGER *Input*
On entry: indicates whether cross covariances are to be calculated in the routine or supplied in the call to the routine.
 IC = 0 if cross covariances are to be calculated,
 IC \neq 0 if cross covariances are to be supplied.

- 8:** NC — INTEGER *Input*
On entry: the number of cross covariances to be calculated in the routine or supplied in the call to the routine.
Constraint: $MW + |IS| \leq NC \leq NXY$.
- 9:** CXY(NC) — *real* array *Input/Output*
On entry: if $IC \neq 0$, then CXY must contain the NC cross covariances between values in the y series and earlier values in time in the x series, for lags from 0 to $(NC - 1)$. If $IC = 0$ CXY need not be set.
On exit: if $IC = 0$, CYX will contain the NC calculated cross covariances.
 If $IC \neq 0$, the contents of CXY will be unchanged.
- 10:** CYX(NC) — *real* array *Input/Output*
On entry: if $IC \neq 0$, then CYX must contain the NC cross covariances between values in the y series and later values in time in the x series, for lags from 0 to $(NC - 1)$. If $IC = 0$, CYX need not be set.
On exit: if $IC = 0$, CYX will contain the NC calculated cross covariances.
 If $IC \neq 0$, the contents of CYX will be unchanged.
- 11:** KC — INTEGER *Input*
On entry: if $IC = 0$, KC must specify the order of the fast Fourier transform (FFT) used to calculate the cross covariances. KC should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq n + NC$.
 If $IC \neq 0$, that is if covariances are supplied, then KC is not used.
Constraints:

$$KC \geq NXY + NC.$$
 The largest prime factor of KC must not exceed 19, and the total number of prime factors of KC, counting repetitions, must not exceed 20. These two restrictions are imposed by C06EAF and C06EBF which perform the FFT.
- 12:** L — INTEGER *Input*
On entry: the frequency division, L of the spectral estimates as $\frac{2\pi}{L}$. Therefore it is also the order of the FFT used to construct the sample spectrum from the cross covariances. L should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq 2M - 1$.
Constraints:

$$L \geq 2 \times MW - 1.$$
 The largest prime factor of L must not exceed 19, and the total number of prime factors of L, counting repetitions, must not exceed 20. These two restrictions are imposed by C06EAF which performs the FFT.
- 13:** NXYG — INTEGER *Input*
On entry: the length of the smaller of the arrays XG and YG, as declared in the (sub)program from which G13CCF is called.
Constraints:
 if $IC = 0$, that is cross covariances are to be calculated, $NXYG \geq \max(KC, L)$,
 if $IC \neq 0$, that is cross covariances are to be supplied, $NXYG \geq L$.

- 14:** XG(NXYG) — *real* array *Input/Output*
On entry: if the cross covariances are to be calculated, then XG must contain the NXY data points of the x series. If covariances are supplied, XG need not be set.
On exit: contains the real parts of the NG complex spectral estimates in elements XG(1) to XG(NG), and XG(NG + 1) to XG(NXYG) contain 0.0. The y series leads the x series.
- 15:** YG(NXYG) — *real* array *Input/Output*
On entry: if cross covariances are to be calculated, then YG must contain the NXY data points of the y series. If covariances are supplied, YG need not be set.
On exit: contains the imaginary parts of the NG complex spectral estimates in elements YG(1) to YG(NG), and YG(NG + 1) to YG(NXYG) contain 0.0. The y series leads the x series.
- 16:** NG — INTEGER *Output*
On exit: the number, $[L/2] + 1$, of complex spectral estimates, whose separate parts are held in XG and YG.
- 17:** IFAIL — INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.
On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors detected by the routine:

IFAIL = 1

- On entry, NXY < 1,
 or MTXY < 0 and IC = 0,
 or MTXY > 2 and IC = 0,
 or PXY < 0.0 and IC = 0,
 or PXY > 1.0 and IC = 0,
 or IW ≤ 0,
 or IW > 4,
 or MW < 1,
 or MW + |IS| > NXY,
 or |IS| ≥ MW,
 or NC < MW + |IS|,
 or NC > NXY,
 or NXYG < max(|rmKC, L) and IC = 0,
 or NXYG < L and IC ≠ 0.

IFAIL = 2

- On entry, KC < NXY + NC,
 or KC has a prime factor exceeding 19,
 or KC has more than 20 prime factors, counting repetitions.

This error only occurs when IC = 0.

IFAIL = 3

- On entry, L < 2 × MW - 1,
 or L has a prime factor exceeding 19,
 or L has more than 20 prime factors, counting repetitions.

7 Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

8 Further Comments

G13CCF carries out two FFTs of length KC by calls to C06EAF and C06EBF to calculate the sample cross covariances and one FFT of length L to calculate the sample spectrum. The timing of G13CCF is therefore dependent on the choice of these values. The time taken for an FFT of length n is approximately proportional to $n \log n$ (but see Section 8 of the document for C06EAF for further details).

9 Example

The example program reads 2 time series of length 296. It then selects mean correction, a 10% tapering proportion, the Parzen smoothing window and a cut-off point of 35 for the lag window. The alignment shift is set to 3 and 50 cross covariances are chosen to be calculated. The program then calls G13CCF to calculate the cross spectrum and then prints the cross covariances and cross spectrum.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```

*      G13CCF Example Program Text
*      Mark 14 Revised.  NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NXYG, NCMAX
      PARAMETER       (NXYG=350,NCMAX=50)
      INTEGER          NIN, NOUT
      PARAMETER       (NIN=5,NOUT=6)
*      .. Local Scalars ..
      real            PXY
      INTEGER          I, IC, IFAIL, II, IS, IW, KC, L, MTXY, MW, NC,
+      NG, NXY
*      .. Local Arrays ..
      real            CXY(NCMAX), CYX(NCMAX), XG(NXYG), YG(NXYG)
*      .. External Subroutines ..
      EXTERNAL        G13CCF
*      .. Intrinsic Functions ..
      INTRINSIC       MIN
*      .. Executable Statements ..
      WRITE (NOUT,*) 'G13CCF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) NXY, NC, IC
      IF (NXY.GT.0 .AND. NXY.LE.NXYG .AND. NC.GT.0 .AND. NC.LE.NCMAX)
+      THEN
          IF (IC.EQ.0) THEN
              READ (NIN,*) (XG(I),I=1,NXY)
              READ (NIN,*) (YG(I),I=1,NXY)
          ELSE
              READ (NIN,*) (CXY(I),I=1,NC)
              READ (NIN,*) (CYX(I),I=1,NC)
          END IF
*      Set parameters for call to G13CCF
*      Mean correction and 10 percent taper

```

```

      MTXY = 1
      PXY = 0.1e0
*      Parzen window and zero covariance at lag 35
      IW = 4
      MW = 35
*      Alignment shift of 3, 50 covariances to be calculated
      IS = 3
      KC = 350
      L = 80
      IFAIL = 0
*
      CALL G13CCF(NXY,MTXY,PXY,IW,MW,IS,IC,NC,CXY,CYX,KC,L,NXYG,XG,
+              YG,NG,IFAIL)
*
      WRITE (NOUT,*)
      WRITE (NOUT,*) '                      Returned cross covariances'
      WRITE (NOUT,*)
      WRITE (NOUT,*)
+ 'Lag      XY      YX      Lag      XY      YX      Lag      XY      YX'
      DO 20 I = 1, NC, 3
          WRITE (NOUT,99999) (II-1,CXY(II),CYX(II),II=I,MIN(I+2,NC))
20      CONTINUE
      WRITE (NOUT,*)
      WRITE (NOUT,*) '                      Returned sample spectrum'
      WRITE (NOUT,*)
      WRITE (NOUT,*)
+ '          Real Imaginary          Real Imaginary          Real Imaginar
+y'
      WRITE (NOUT,*)
+ 'Lag      part      part Lag      part      part Lag      part      part'
      DO 40 I = 1, NG, 3
          WRITE (NOUT,99999) (II-1,XG(II),YG(II),II=I,MIN(I+2,NG))
40      CONTINUE
      END IF
      STOP
*
99999 FORMAT (1X,I3,2F9.4,I4,2F9.4,I4,2F9.4)
      END

```

9.2 Program Data

G13CCF Example Program Data

```

296 50 0
-0.109 0.000 0.178 0.339 0.373 0.441 0.461 0.348
 0.127 -0.180 -0.588 -1.055 -1.421 -1.520 -1.302 -0.814
-0.475 -0.193 0.088 0.435 0.771 0.866 0.875 0.891
 0.987 1.263 1.775 1.976 1.934 1.866 1.832 1.767
 1.608 1.265 0.790 0.360 0.115 0.088 0.331 0.645
 0.960 1.409 2.670 2.834 2.812 2.483 1.929 1.485
 1.214 1.239 1.608 1.905 2.023 1.815 0.535 0.122
 0.009 0.164 0.671 1.019 1.146 1.155 1.112 1.121
 1.223 1.257 1.157 0.913 0.620 0.255 -0.280 -1.080
-1.551 -1.799 -1.825 -1.456 -0.944 -0.570 -0.431 -0.577
-0.960 -1.616 -1.875 -1.891 -1.746 -1.474 -1.201 -0.927
-0.524 0.040 0.788 0.943 0.930 1.006 1.137 1.198
 1.054 0.595 -0.080 -0.314 -0.288 -0.153 -0.109 -0.187
-0.255 -0.299 -0.007 0.254 0.330 0.102 -0.423 -1.139
-2.275 -2.594 -2.716 -2.510 -1.790 -1.346 -1.081 -0.910

```

```

-0.876 -0.885 -0.800 -0.544 -0.416 -0.271 0.000 0.403
0.841 1.285 1.607 1.746 1.683 1.485 0.993 0.648
0.577 0.577 0.632 0.747 0.999 0.993 0.968 0.790
0.399 -0.161 -0.553 -0.603 -0.424 -0.194 -0.049 0.060
0.161 0.301 0.517 0.566 0.560 0.573 0.592 0.671
0.933 1.337 1.460 1.353 0.772 0.218 -0.237 -0.714
-1.099 -1.269 -1.175 -0.676 0.033 0.556 0.643 0.484
0.109 -0.310 -0.697 -1.047 -1.218 -1.183 -0.873 -0.336
0.063 0.084 0.000 0.001 0.209 0.556 0.782 0.858
0.918 0.862 0.416 -0.336 -0.959 -1.813 -2.378 -2.499
-2.473 -2.330 -2.053 -1.739 -1.261 -0.569 -0.137 -0.024
-0.050 -0.135 -0.276 -0.534 -0.871 -1.243 -1.439 -1.422
-1.175 -0.813 -0.634 -0.582 -0.625 -0.713 -0.848 -1.039
-1.346 -1.628 -1.619 -1.149 -0.488 -0.160 -0.007 -0.092
-0.620 -1.086 -1.525 -1.858 -2.029 -2.024 -1.961 -1.952
-1.794 -1.302 -1.030 -0.918 -0.798 -0.867 -1.047 -1.123
-0.876 -0.395 0.185 0.662 0.709 0.605 0.501 0.603
0.943 1.223 1.249 0.824 0.102 0.025 0.382 0.922
1.032 0.866 0.527 0.093 -0.458 -0.748 -0.947 -1.029
-0.928 -0.645 -0.424 -0.276 -0.158 -0.033 0.102 0.251
0.280 0.000 -0.493 -0.759 -0.824 -0.740 -0.528 -0.204
0.034 0.204 0.253 0.195 0.131 0.017 -0.182 -0.262
53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0 52.4 53.0 54.0 54.9 56.0
56.8 56.8 56.4 55.7 55.0 54.3 53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9
47.6 47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2 50.0 48.3 47.0 45.8
45.6 46.0 46.9 47.8 48.2 48.3 47.9 47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5
50.1 49.8 49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0 56.9 57.5 57.3
56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1
51.6 51.0 50.5 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8 53.3 53.0
52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4 58.4 57.6 56.9 56.4 56.0 55.7 55.3
55.0 54.4 53.7 52.8 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9 50.7
50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0 53.6 53.2 53.0 52.8 52.3 51.9
51.6 51.6 51.4 51.2 50.7 50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9 55.2 54.4 53.7 53.6 53.6
53.2 52.5 52.0 51.4 51.0 50.9 52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7
59.0 57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3 57.4 57.0 56.4 55.9
55.5 55.3 55.2 55.4 56.0 56.5 57.1 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2
57.8 58.3 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4 56.0 55.2 54.0
53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0 52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2
52.0 52.8 53.8 54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0 54.3 56.0
57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0

```

9.3 Program Results

G13CCF Example Program Results

Returned cross covariances

Lag	XY	YX	Lag	XY	YX	Lag	XY	YX
0	-1.6700	-1.6700	1	-2.0581	-1.3606	2	-2.4859	-1.1383
3	-2.8793	-0.9926	4	-3.1473	-0.9009	5	-3.2239	-0.8382
6	-3.0929	-0.7804	7	-2.7974	-0.7074	8	-2.4145	-0.6147
9	-2.0237	-0.5080	10	-1.6802	-0.4032	11	-1.4065	-0.3159
12	-1.2049	-0.2554	13	-1.0655	-0.2250	14	-0.9726	-0.2238
15	-0.9117	-0.2454	16	-0.8658	-0.2784	17	-0.8180	-0.3081
18	-0.7563	-0.3257	19	-0.6750	-0.3315	20	-0.5754	-0.3321
21	-0.4701	-0.3308	22	-0.3738	-0.3312	23	-0.3023	-0.3332
24	-0.2665	-0.3384	25	-0.2645	-0.3506	26	-0.2847	-0.3727
27	-0.3103	-0.3992	28	-0.3263	-0.4152	29	-0.3271	-0.4044
30	-0.3119	-0.3621	31	-0.2837	-0.2919	32	-0.2568	-0.2054
33	-0.2427	-0.1185	34	-0.2490	-0.0414	35	-0.2774	0.0227
36	-0.3218	0.0697	37	-0.3705	0.1039	38	-0.4083	0.1356
39	-0.4197	0.1805	40	-0.3920	0.2460	41	-0.3241	0.3319
42	-0.2273	0.4325	43	-0.1216	0.5331	44	-0.0245	0.6199
45	0.0528	0.6875	46	0.1074	0.7329	47	0.1448	0.7550
48	0.1713	0.7544	49	0.1943	0.7349			

Returned sample spectrum

Lag	Real part	Imaginary part	Lag	Real part	Imaginary part	Lag	Real part	Imaginary part
0	-6.5500	0.0000	1	-5.4267	-1.9842	2	-3.1323	-2.7307
3	-1.2649	-2.3998	4	-0.2102	-1.7520	5	0.3411	-1.1903
6	0.6063	-0.7420	7	0.6178	-0.3586	8	0.4391	-0.1008
9	0.2422	0.0061	10	0.1233	0.0409	11	0.0574	0.0529
12	0.0174	0.0452	13	-0.0008	0.0289	14	-0.0058	0.0161
15	-0.0051	0.0084	16	-0.0027	0.0040	17	-0.0010	0.0015
18	-0.0006	0.0006	19	-0.0005	0.0003	20	-0.0003	0.0003
21	-0.0003	0.0004	22	-0.0003	0.0003	23	-0.0003	0.0002
24	-0.0004	0.0001	25	-0.0004	0.0000	26	-0.0003	-0.0001
27	-0.0002	-0.0001	28	-0.0001	0.0001	29	-0.0002	0.0003
30	-0.0003	0.0002	31	-0.0002	0.0001	32	-0.0001	0.0000
33	0.0000	0.0000	34	0.0001	-0.0001	35	0.0001	-0.0002
36	0.0001	-0.0001	37	0.0001	-0.0001	38	0.0001	-0.0001
39	0.0001	-0.0001	40	0.0001	0.0000			