G13CDF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

G13CDF calculates the smoothed sample cross spectrum of a bivariate time series using spectral smoothing by the trapezium frequency (Daniell) window.

2 Specification

```
 SUBROUTINE G13CDF(NXY, MTXY, PXY, MW, IS, PW, L, KC, XG, YG, NG,
1                  IFAIL)
 INTEGER           NXY, MTXY, MW, IS, L, KC, NG, IFAIL
 real              PXY, PW, XG(KC), YG(KC)
```

3 Description

The supplied time series may be mean and trend corrected and tapered as in the description of G13CBF before calculation of the unsmoothed sample cross-spectrum

$$f_{xy}^{*}(\omega) = \frac{1}{2\pi n}\left\{\sum_{t=1}^{n} y_t \exp(i\omega t)\right\} \times \left\{\sum_{t=1}^{n} x_t \exp(-i\omega t)\right\}$$

for frequency values $\omega_j = \frac{2\pi j}{K}$, $0 \leq \omega_j \leq \pi$.

A correction is made for bias due to any tapering.

As in the description of G13CBF for univariate frequency window smoothing, the smoothed spectrum is returned as a subset of these frequencies,

$$\nu_l = \frac{2\pi l}{L}\ ,\ l = 0, 1, \ldots, [L/2]$$

where [ ] denotes the integer part.

Its real part or co-spectrum $cf(\nu_l)$, and imaginary part or quadrature spectrum $qf(\nu_l)$ are defined by

$$f_{xy}(\nu_l) = cf(\nu_l) + iqf(\nu_l) = \sum_{|\omega_k| < \frac{\pi}{M}} \tilde{w}_k f_{xy}^{*}(\nu_l + \omega_k)$$

where the weights $\tilde{w}_k$ are similar to the weights w_k defined for G13CBF, but allow for an implicit alignment shift S between the series:

$$\tilde{w}_k = w_k \exp(-2\pi i S k/L).$$

It is recommended that S is chosen as the lag k at which the cross covariances $c_{xy}(k)$ peak, so as to minimize bias.

If no smoothing is required, the integer M which determines the frequency window width $\frac{2\pi}{M}$, should be set to n.

The bandwidth of the estimates will normally have been calculated in a previous call of G13CBF for estimating the univariate spectra of y_t and x_t.

4 References

[1] Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden–Day

[2] Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

5 Parameters

1: NXY — INTEGER *Input*

On entry: the length of the time series x and y, n.

Constraint: NXY ≥ 1.

2: MTXY — INTEGER *Input*

On entry: whether the data is to be initially mean or trend corrected.

MTXY $= 0$ for no correction,

MTXY $= 1$ for mean correction,

MTXY $= 2$ for trend correction.

Constraint: $0 \leq$ MTXY ≤ 2.

3: PXY — ***real*** *Input*

On entry: the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper.

A value of 0.0 implies no tapering.

Constraint: $0.0 \leq$ PXY ≤ 1.0.

4: MW — INTEGER *Input*

On entry: the frequency width, M, of the smoothing window as $\frac{2\pi}{M}$.

A value of n implies that no smoothing is to be carried out.

Constraint: $1 \leq$ MW $\leq$ NXY.

5: IS — INTEGER *Input*

On entry: the alignment shift, S, between the x and y series. If x leads y, the shift is positive.

Constraint: $-$L $<$ IS $<$ L.

6: PW — ***real*** *Input*

On entry: the shape parameter, p, of the trapezium frequency window.

A value of 0.0 gives a triangular window, and a value of 1.0 a rectangular window.

If MW = NXY (i.e., no smoothing is carried out) then PW is not used.

Constraint: $0.0 \leq$ PW ≤ 1.0 if MW $\neq$ NXY.

7: L — INTEGER *Input*

On entry: the frequency division, L, of smoothed cross spectral estimates as $\frac{2\pi}{L}$.

Constraints:

L ≥ 1,
L must be a factor of KC (see below).

8: KC — INTEGER *Input*

On entry: the order of the fast Fourier transform (FFT) used to calculate the spectral estimates. KC should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq 2n$, provided $m \leq 20$.

Constraints:

KC $\geq 2 \times$ NXY,
KC must be a multiple of L. The largest prime factor of KC must not exceed 19, and the total number of prime factors of KC, counting repetitions, must not exceed 20. These two restrictions are imposed by C06EAF and C06EBF which perform the FFT.

9: XG(KC) — ***real*** array *Input/Output*

On entry: the NXY data points of the x series.

On exit: the real parts of the NG cross spectral estimates in elements XG(1) to XG(NG), and XG(NG + 1) to XG(KC) contain 0.0. The y series leads the x series.

10: YG(KC) — ***real*** array *Input/Output*

On entry: the NXY data points of the y series.

On exit: the imaginary parts of the NG cross spectral estimates in elements YG(1) to YG(NG), and YG(NG + 1) to YG(KC) contain 0.0. The y series leads the x series.

11: NG — INTEGER *Output*

On exit: the number of spectral estimates, $[L/2]+1$, whose separate parts are held in XG and YG.

12: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors detected by the routine:

IFAIL = 1

On entry, NXY $<$ 1,
or MTXY $<$ 0,
or MTXY $>$ 2,
or PXY $<$ 0.0,
or PXY $>$ 1.0,
or MW $<$ 1,
or MW $>$ NXY,
or PW $<$ 0.0 and MW $\neq$ NXY,
or PW $>$ 1.0 and MW $\neq$ NXY,
or L $<$ 1,
or $|$IS$| \geq$ L.

IFAIL = 2

On entry, KC $< 2 \times$ NXY,
or KC is not a multiple of L,
or KC has a prime factor exceeding 19,

or KC has more than 20 prime factors, counting repetitions.

IFAIL = 3

This indicates that a serious error has occurred. Check all array subscripts in calls to G13CDF. Seek expert help.

7 Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

8 Further Comments

G13CDF carries out an FFT of length KC to calculate the sample cross spectrum. The time taken by the routine for this is approximately proportional to KC $\times$ log(KC) (but see routine document C06EAF for further details).

9 Example

The example program reads 2 time series of length 296. It selects mean correction and a 10% tapering proportion. It selects a $2\pi/16$ frequency width of smoothing window, a window shape parameter of 0.5 and an alignment shift of 3. It then calls G13CDF to calculate the smoothed sample cross spectrum and prints the results.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G13CDF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NXYMAX, L, KC
      PARAMETER        (NXYMAX=300,L=80,KC=8*L)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             PW, PXY
      INTEGER          I, IFAIL, IS, J, MTXY, MW, NG, NXY
*     .. Local Arrays ..
      real             XG(KC), YG(KC)
*     .. External Subroutines ..
      EXTERNAL         G13CDF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G13CDF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) NXY
      IF (NXY.GT.0 .AND. NXY.LE.NXYMAX) THEN
         READ (NIN,*) (XG(I),I=1,NXY)
         READ (NIN,*) (YG(I),I=1,NXY)
*        Set parameters for call to G13CDF
*        Mean correction and 10 percent taper
         MTXY = 1
         PXY = 0.1e0
*        Window shape parameter and zero covariance at lag 16
```

```
         PW = 0.5e0
         MW = 16
*        Alignment shift of 3
         IS = 3
         IFAIL = 0
*
         CALL G13CDF(NXY,MTXY,PXY,MW,IS,PW,L,KC,XG,YG,NG,IFAIL)
*
         WRITE (NOUT,*)
         WRITE (NOUT,*) '                      Returned sample spectrum'
         WRITE (NOUT,*)
         WRITE (NOUT,*)
     +'     Real  Imaginary      Real  Imaginary      Real  Imaginary
     +'
         WRITE (NOUT,*)
     + '      part     part         part     part         part     part'
         WRITE (NOUT,99999) (J,XG(J),YG(J),J=1,NG)
      END IF
      STOP
*
99999 FORMAT (1X,I3,F8.4,F9.4,I5,F8.4,F9.4,I5,F8.4,F9.4)
      END
```

9.2 Program Data

```
G13CDF Example Program Data
   296
 -0.109  0.000  0.178  0.339  0.373  0.441  0.461  0.348
  0.127 -0.180 -0.588 -1.055 -1.421 -1.520 -1.302 -0.814
 -0.475 -0.193  0.088  0.435  0.771  0.866  0.875  0.891
  0.987  1.263  1.775  1.976  1.934  1.866  1.832  1.767
  1.608  1.265  0.790  0.360  0.115  0.088  0.331  0.645
  0.960  1.409  2.670  2.834  2.812  2.483  1.929  1.485
  1.214  1.239  1.608  1.905  2.023  1.815  0.535  0.122
  0.009  0.164  0.671  1.019  1.146  1.155  1.112  1.121
  1.223  1.257  1.157  0.913  0.620  0.255 -0.280 -1.080
 -1.551 -1.799 -1.825 -1.456 -0.944 -0.570 -0.431 -0.577
 -0.960 -1.616 -1.875 -1.891 -1.746 -1.474 -1.201 -0.927
 -0.524  0.040  0.788  0.943  0.930  1.006  1.137  1.198
  1.054  0.595 -0.080 -0.314 -0.288 -0.153 -0.109 -0.187
 -0.255 -0.299 -0.007  0.254  0.330  0.102 -0.423 -1.139
 -2.275 -2.594 -2.716 -2.510 -1.790 -1.346 -1.081 -0.910
 -0.876 -0.885 -0.800 -0.544 -0.416 -0.271  0.000  0.403
  0.841  1.285  1.607  1.746  1.683  1.485  0.993  0.648
  0.577  0.577  0.632  0.747  0.999  0.993  0.968  0.790
  0.399 -0.161 -0.553 -0.603 -0.424 -0.194 -0.049  0.060
  0.161  0.301  0.517  0.566  0.560  0.573  0.592  0.671
  0.933  1.337  1.460  1.353  0.772  0.218 -0.237 -0.714
 -1.099 -1.269 -1.175 -0.676  0.033  0.556  0.643  0.484
  0.109 -0.310 -0.697 -1.047 -1.218 -1.183 -0.873 -0.336
  0.063  0.084  0.000  0.001  0.209  0.556  0.782  0.858
  0.918  0.862  0.416 -0.336 -0.959 -1.813 -2.378 -2.499
 -2.473 -2.330 -2.053 -1.739 -1.261 -0.569 -0.137 -0.024
 -0.050 -0.135 -0.276 -0.534 -0.871 -1.243 -1.439 -1.422
 -1.175 -0.813 -0.634 -0.582 -0.625 -0.713 -0.848 -1.039
 -1.346 -1.628 -1.619 -1.149 -0.488 -0.160 -0.007 -0.092
 -0.620 -1.086 -1.525 -1.858 -2.029 -2.024 -1.961 -1.952
 -1.794 -1.302 -1.030 -0.918 -0.798 -0.867 -1.047 -1.123
```

```
-0.876 -0.395  0.185  0.662  0.709  0.605  0.501  0.603
 0.943  1.223  1.249  0.824  0.102  0.025  0.382  0.922
 1.032  0.866  0.527  0.093 -0.458 -0.748 -0.947 -1.029
-0.928 -0.645 -0.424 -0.276 -0.158 -0.033  0.102  0.251
 0.280  0.000 -0.493 -0.759 -0.824 -0.740 -0.528 -0.204
 0.034  0.204  0.253  0.195  0.131  0.017 -0.182 -0.262
53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0 52.4 53.0 54.0 54.9 56.0
56.8 56.8 56.4 55.7 55.0 54.3 53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9
47.6 47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2 50.0 48.3 47.0 45.8
45.6 46.0 46.9 47.8 48.2 48.3 47.9 47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5
50.1 49.8 49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0 56.9 57.5 57.3
56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1
51.6 51.0 50.5 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8 53.3 53.0
52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4 58.4 57.6 56.9 56.4 56.0 55.7 55.3
55.0 54.4 53.7 52.8 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9 50.7
50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0 53.6 53.2 53.0 52.8 52.3 51.9
51.6 51.6 51.4 51.2 50.7 50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9 55.2 54.4 53.7 53.6 53.6
53.2 52.5 52.0 51.4 51.0 50.9 52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7
59.0 57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3 57.4 57.0 56.4 55.9
55.5 55.3 55.2 55.4 56.0 56.5 57.1 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2
57.8 58.3 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4 56.0 55.2 54.0
53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0 52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2
52.0 52.8 53.8 54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0 54.3 56.0
57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0
```

9.3 Program Results

```
G13CDF Example Program Results

                     Returned sample spectrum

      Real  Imaginary       Real  Imaginary       Real  Imaginary
      part     part         part     part         part     part
  1 -6.1563   0.0000    2 -5.5905  -2.0119    3 -3.2711  -2.7963
  4 -1.1803  -2.3264    5 -0.2061  -1.8132    6  0.3434  -1.1357
  7  0.6200  -0.7351    8  0.5967  -0.3449    9  0.4523  -0.0984
 10  0.2391   0.0177   11  0.1129   0.0402   12  0.0564   0.0523
 13  0.0134   0.0443   14 -0.0032   0.0299   15 -0.0057   0.0148
 16 -0.0057   0.0069   17 -0.0033   0.0038   18 -0.0011   0.0012
 19 -0.0004   0.0001   20 -0.0004   0.0002   21 -0.0003   0.0001
 22 -0.0001   0.0002   23 -0.0002   0.0003   24 -0.0002   0.0002
 25 -0.0002   0.0000   26 -0.0004   0.0000   27 -0.0002  -0.0002
 28 -0.0001   0.0000   29 -0.0001   0.0002   30 -0.0001   0.0002
 31 -0.0002   0.0003   32 -0.0002   0.0001   33 -0.0001   0.0000
 34  0.0000   0.0000   35  0.0000  -0.0001   36  0.0001  -0.0001
 37  0.0001  -0.0001   38  0.0001  -0.0001   39  0.0000  -0.0001
 40  0.0000  -0.0001   41  0.0001   0.0000
```