S07AAF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

S07AAF returns the value of the circular tangent, $\tan x$, via the routine name.

2 Specification

```
real FUNCTION S07AAF(X, IFAIL)
INTEGER              IFAIL
real                 X
```

3 Description

The routine calculates an approximate value for the circular tangent of its argument, $\tan x$. It is based on the Chebyshev expansion

$$\tan\theta = \theta y(t) = \theta \sum_{r=0}{}' c_r T_r(t)$$

where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$ and $-1 < t < +1$, $\quad t = 2\left(\frac{4\theta}{\pi}\right)^2 - 1$.

The reduction to the standard range is accomplished by taking

$$x = N\pi/2 + \theta$$

where N is an integer and $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$,

i.e., $\theta = x - \left(\frac{2x}{\pi}\right)\frac{\pi}{2}$ where $N = \left[\frac{2x}{\pi}\right] =$ the nearest integer to $\frac{2x}{\pi}$.

From the properties of $\tan x$ it follows that

$$\tan x = \left\{ \begin{array}{rl} \tan\theta, & N \text{ even} \\ -1/\tan\theta, & N \text{ odd} \end{array} \right\}$$

4 References

[1] Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

5 Parameters

1: X — ***real*** *Input*

On entry: the argument x of the function.

2: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, −1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors detected by the routine:

IFAIL = 1

The routine has been called with an argument that is too large; the default result returned is zero.

IFAIL = 2

The routine has been called with an argument that is too close to an odd multiple of $\pi/2$, at which the function is infinite; the routine returns a value with the correct sign but a more or less arbitrary but large magnitude (see Section 7).

7 Accuracy

If δ and ϵ are the relative errors in the argument and result respectively, then in principle

$$\epsilon \geq \frac{2x}{\sin 2x}\delta.$$

That is a relative error in the argument, x, is amplified by at least a factor $2x/\sin 2x$ in the result.

Similarly if E is the absolute error in the result this is given by

$$E \geq \frac{x}{\cos^2 x}\delta.$$

The equalities should hold if δ is greater than the ***machine precision*** (δ is a result of data errors etc.) but if δ is simply the round-off error in the machine it is possible that internal calculation rounding will lose an extra figure.

The graphs below show the behaviour of these amplification factors.

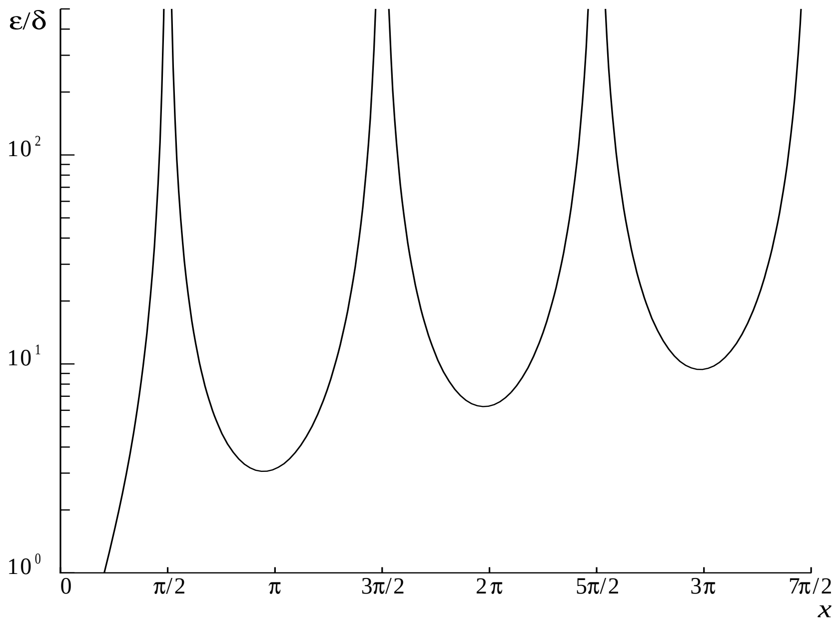

Figure 1

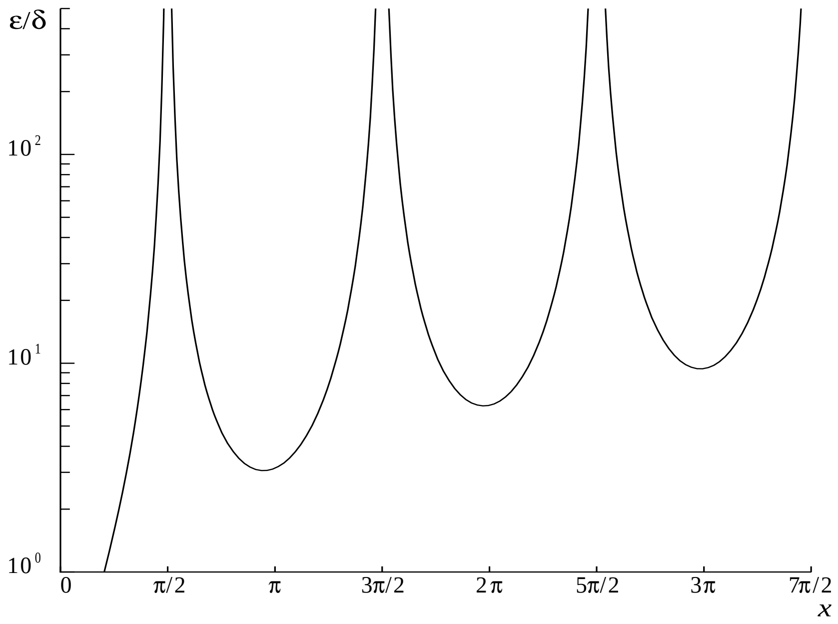

Figure 2

In the principal range it is possible to preserve relative accuracy even near the zero of $\tan x$ at $x = 0$ but at the other zeros only absolute accuracy is possible. Near the infinities of $\tan x$ both the relative and absolute errors become infinite and the routine must fail (error 2).

If N is odd and $|\theta| \leq xF_2$ the routine could not return better than two figures and in all probability would produce a result that was in error in its most significant figure. Therefore the routine fails and it returns the value

$$-\operatorname{sign}\theta\left(\frac{1}{|xF_2|}\right) \simeq -\operatorname{sign}\theta\tan\left(\frac{\pi}{2} - |xF_2|\right)$$

which is the value of the tangent at the nearest argument for which a valid call could be made.

Accuracy is also unavoidably lost if the routine is called with a large argument. If $|x| > F_1$ the routine fails (error 1) and returns zero. (See the Users' Note for your implementation for specific values of F_1 and F_2).

8 Further Comments

None.

9 Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S07AAF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
```

```
      real             X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      real             S07AAF
      EXTERNAL         S07AAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S07AAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y         IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S07AAF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y, IFAIL
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,1P,2e12.3,I7)
      END
```

9.2 Program Data

```
S07AAF Example Program Data
            -2.0
            -0.5
             1.0
             3.0
             1.5708
```

9.3 Program Results

```
 S07AAF Example Program Results

      X           Y         IFAIL

   -2.000E+00   2.185E+00      0
   -5.000E-01  -5.463E-01      0
    1.000E+00   1.557E+00      0
    3.000E+00  -1.425E-01      0
    1.571E+00  -2.722E+05      0
```