S09AAF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

S09AAF returns the value of the inverse circular sine, $\arcsin x$, via the routine name. The value is in the principal range $(-\pi/2, \pi/2)$.

2 Specification

```
real FUNCTION S09AAF(X, IFAIL)
INTEGER              IFAIL
real                 X
```

3 Description

The routine calculates an approximate value for the inverse circular sine, $\arcsin x$. It is based on the Chebyshev expansion

$$\arcsin x = x \times y(x) = x \sum_{r=0}^{\prime} a_r T_r(t)$$

where $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$ and $t = 4x^2 - 1$.

For $x^2 \leq \frac{1}{2}$, $\arcsin x = x \times y(x)$.

For $\frac{1}{2} < x^2 \leq 1$, $\arcsin x = \operatorname{sign} x \left\{ \frac{\pi}{2} - \arcsin \sqrt{1 - x^2} \right\}$.

For $x^2 > 1$, $\arcsin x$ is undefined and the routine fails.

4 References

[1] Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

5 Parameters

1: X — ***real*** *Input*

On entry: the argument x of the function.

Constraint: $|\mathrm{X}| \leq 1.0$.

2: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors detected by the routine:

IFAIL $= 1$

The routine has been called with an argument greater than 1.0 in absolute value; $\arcsin x$ is undefined and the routine returns zero.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\arcsin x \sqrt{1-x^2}} \times \delta \right|.$$

That is, a relative error in the argument x, is amplified by at least a factor $\frac{x}{\arcsin x \sqrt{1-x^2}}$ in the result.

The equality should hold if δ is greater than the ***machine precision*** (δ is a result of data errors etc.) but if δ is produced simply by round-off error in the machine it is possible that rounding in internal calculations may lose an extra figure in the result.

This factor stays close to one except near $|x| = 1$ where its behaviour is shown in the following graph.

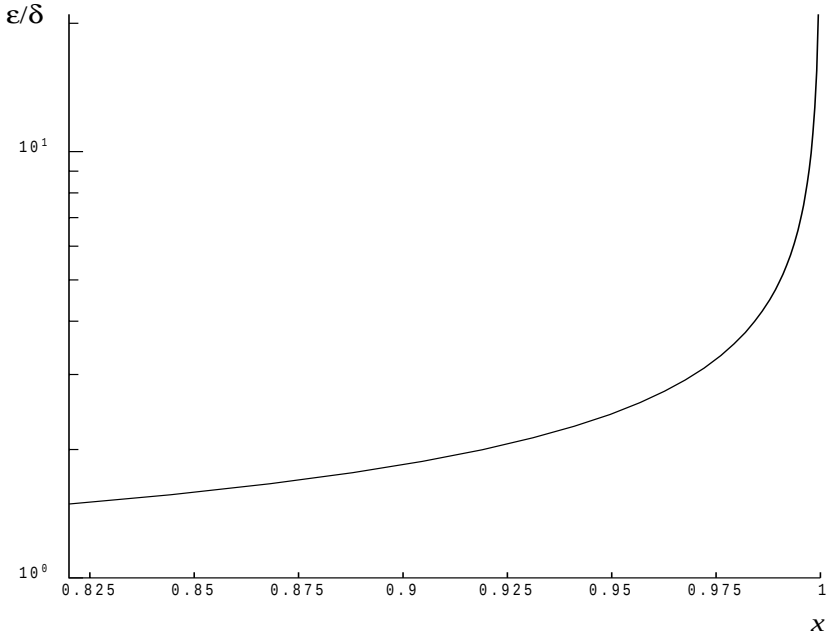

Figure 1

For $|x|$ close to unity, $1 - |x| \sim \delta$, the above analysis is no longer applicable owing to the fact that both argument and result are subject to finite bounds, ($|x| \leq 1$ and $|\arcsin x| \leq \frac{1}{2}\pi$). In this region $\epsilon \sim \sqrt{\delta}$; that is the result will have approximately half as many correct significant figures as the argument.

For $|x| = 1$ the result will be correct to full ***machine precision***.

8 Further Comments

None.

9 Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S09AAF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      real             S09AAF
      EXTERNAL         S09AAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S09AAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S09AAF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y, IFAIL
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,1P,2e12.3,I7)
      END
```

9.2 Program Data

```
S09AAF Example Program Data
           -0.5
            0.1
            0.9
            2.0
           -1.5
```

9.3 Program Results

```
 S09AAF Example Program Results

      X           Y        IFAIL

  -5.000E-01  -5.236E-01      0
   1.000E-01   1.002E-01      0
   9.000E-01   1.120E+00      0
   2.000E+00   0.000E+00      1
  -1.500E+00   0.000E+00      1
```