S10AAF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

S10AAF returns a value for the hyperbolic tangent, $\tanh x$, via the routine name.

2 Specification

```
real FUNCTION S10AAF(X, IFAIL)
INTEGER              IFAIL
real                 X
```

3 Description

The routine calculates an approximate value for the hyperbolic tangent of its argument, $\tanh x$.

For $|x| \leq 1$ it is based on the Chebyshev expansion

$$\tanh x = x \times y(t) = x \sum_{r=0}{}' a_r T_r(t)$$

where $-1 \leq x \leq 1$, $\quad -1 \leq t \leq 1$, $\quad$ and $t = 2x^2 - 1$.

For $1 < |x| < E_1$ (See the Users' Note for your implementation for value of E_1)

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}$$

For $|x| \geq E_1$, $\tanh x = \operatorname{sign} x$ to within the representation accuracy of the machine and so this approximation is used.

4 References

[1] Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

5 Parameters

1: X — ***real*** *Input*

On entry: the argument x of the function.

2: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors detected by the routine:

There are no error exits from this routine. The parameter IFAIL is included for consistency with the other routines in this chapter.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result respectively, then in principle,

$$|\epsilon| \simeq \left| \frac{2x}{\sinh 2x} \delta \right|.$$

That is, a relative error in the argument, x, is amplified by a factor approximately $\frac{2x}{\sinh 2x}$, in the result.

The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is due simply to the round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

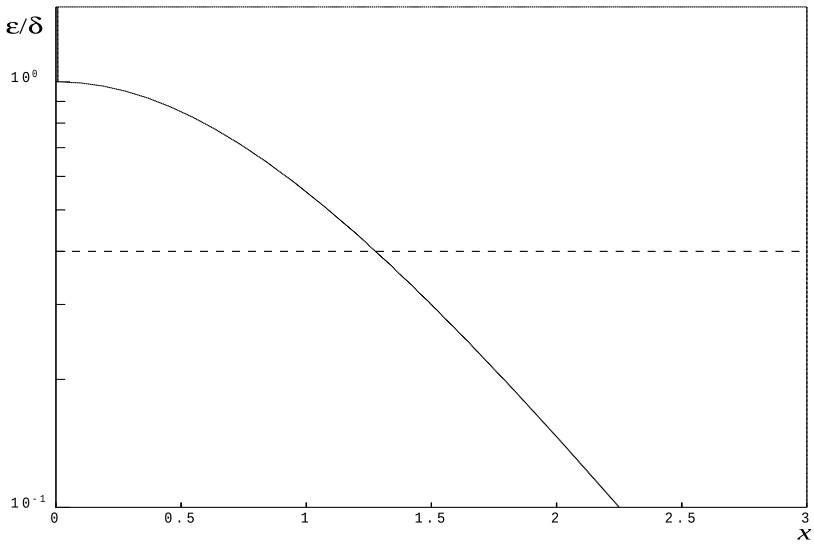

Figure 1

It should be noted that this factor is always less than or equal to 1.0 and away from $x = 0$ the accuracy will eventually be limited entirely by the precision of machine representation.

8 Further Comments

None.

9 Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S10AAF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
```

```
      real             X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      real             S10AAF
      EXTERNAL         S10AAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S10AAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S10AAF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y, IFAIL
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,1P,2e12.3,I7)
      END
```

9.2 Program Data

```
S10AAF Example Program Data
           -20.0
            -5.0
             0.5
             5.0
```

9.3 Program Results

```
 S10AAF Example Program Results

      X           Y        IFAIL

   -2.000E+01  -1.000E+00      0
   -5.000E+00  -9.999E-01      0
    5.000E-01   4.621E-01      0
    5.000E+00   9.999E-01      0
```