S10ABF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

S10ABF returns the value of the hyperbolic sine, $\sinh x$, via the routine name.

2 Specification

```
real FUNCTION S10ABF(X, IFAIL)
INTEGER              IFAIL
real                 X
```

3 Description

The routine calculates an approximate value for the hyperbolic sine of its argument, $\sinh x$.

For $|x| \leq 1$ it uses the Chebyshev expansion

$$\sinh x = x \times y(t) = x \sum_{r=0}{}' a_r T_r(t)$$

where $t = 2x^2 - 1$.

For $1 < |x| \leq E_1$, $\quad \sinh x = \frac{1}{2}(e^x - e^{-x})$

where E_1 is a machine-dependent constant, details of which are given in the Users' Note for your implementation.

For $|x| > E_1$, the routine fails owing to the danger of setting overflow in calculating e^x. The result returned for such calls is $\sinh(\operatorname{sign} x E_1)$, i.e., it returns the result for the nearest valid argument.

4 References

[1] Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

5 Parameters

1: X — ***real*** *Input*

On entry: the argument x of the function.

2: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors detected by the routine:

IFAIL = 1

The routine has been called with an argument too large in absolute magnitude. There is a danger of setting overflow. The result is the value of sinh at the closest argument for which a valid call could be made. (See Section 3 and the Users' Note for your implementation).

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq |x \coth x \times \delta|.$$

That is the relative error in the argument, x, is amplified by a factor, approximately $x \coth x$. The equality should hold if δ is greater than the ***machine precision*** (δ is a result of data errors etc.) but, if δ is simply a result of round-off in the machine representation of x, then it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the error amplification factor can be seen in the following graph:

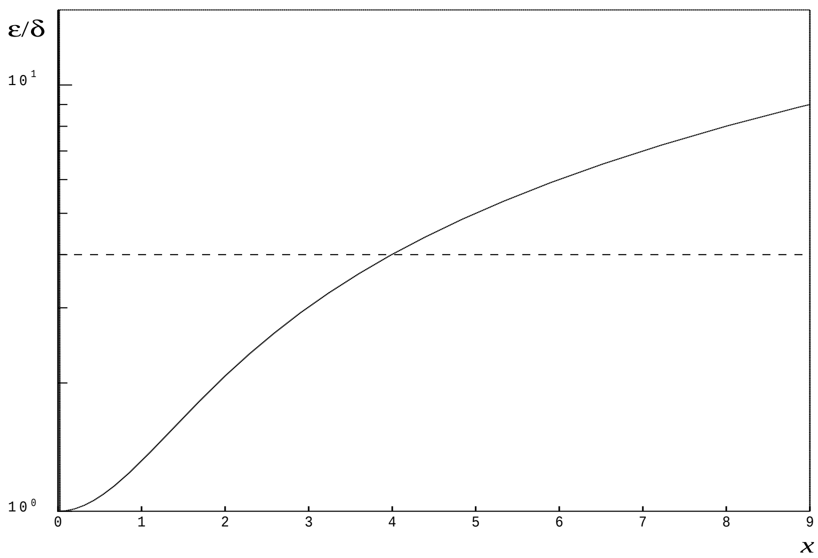

Figure 1

It should be noted that for $|x| \geq 2$

$$\epsilon \sim x\delta = \Delta$$

where Δ is the absolute error in the argument.

8 Further Comments

None.

9 Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S10ABF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
```

```
      real             X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      real             S10ABF
      EXTERNAL         S10ABF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S10ABF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S10ABF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y, IFAIL
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,1P,2e12.3,I7)
      END
```

9.2 Program Data

```
S10ABF Example Program Data
           -10.0
            -0.5
             0.0
             0.5
            25.0
```

9.3 Program Results

```
 S10ABF Example Program Results

      X           Y        IFAIL

   -1.000E+01  -1.101E+04      0
   -5.000E-01  -5.211E-01      0
    0.000E+00   0.000E+00      0
    5.000E-01   5.211E-01      0
    2.500E+01   3.600E+10      0
```