S13ADF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

S13ADF returns the value of the sine integral

$$\mathrm{Si}(x) = \int_0^x \frac{\sin u}{u} du,$$

via the routine name.

2 Specification

```
real FUNCTION S13ADF(X, IFAIL)
INTEGER              IFAIL
real                 X
```

3 Description

The routine calculates an approximate value for $\mathrm{Si}(x)$.

For $|x| \leq 16.0$ it is based on the Chebyshev expansion

$$\mathrm{Si}(x) = x{\sum_{r=0}}' a_r T_r(t)\ ,\ t = 2\left(\frac{x}{16}\right)^2 - 1.$$

For $16 < |x| < x_{hi}$, where x_{hi} is an implementation-dependent number,

$$\mathrm{Si}(x) = \mathrm{sign(x)}\left\{\frac{\pi}{2} - \frac{f(x)\cos x}{x} - \frac{g(x)\sin x}{x^2}\right\}$$

where $f(x) = {\sum_{r=0}}' f_r T_r(t)$ and $g(x) = {\sum_{r=0}}' g_r T_r(t)$, $t = 2\left(\frac{16}{x}\right)^2 - 1$.

For $|x| \geq x_{hi}$, $\mathrm{Si}(x) = \frac{1}{2}\pi\ \mathrm{sign}\, x$ to within ***machine precision***.

4 References

[1] Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

5 Parameters

1: X — ***real*** *Input*

On entry: the argument x of the function.

2: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, −1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors detected by the routine:

There are no failure exits from this routine. The parameter IFAIL has been included for consistency with other routines in this chapter.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{\delta \sin x}{\mathrm{Si}(x)} \right|.$$

The equality may hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation, then since the factor relating δ to ϵ is always less than one, the accuracy will be limited by ***machine precision***.

8 Further Comments

None.

9 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S13ADF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      real             S13ADF
      EXTERNAL         S13ADF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S13ADF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S13ADF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y, IFAIL
      GO TO 20
   40 STOP
```

```
*
99999 FORMAT (1X,1P,2e12.3,I7)
      END
```

9.2 Program Data

```
S13ADF Example Program Data
             0.0
             0.2
             0.4
             0.6
             0.8
             1.0
```

9.3 Program Results

```
 S13ADF Example Program Results

      X           Y        IFAIL

   0.000E+00   0.000E+00      0
   2.000E-01   1.996E-01      0
   4.000E-01   3.965E-01      0
   6.000E-01   5.881E-01      0
   8.000E-01   7.721E-01      0
   1.000E+00   9.461E-01      0
```