

## S15AEF – NAG Fortran Library Routine Document

**Note.** Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

### 1 Purpose

S15AEF returns the value of the error function  $\operatorname{erf} x$ , via the routine name.

### 2 Specification

```
real FUNCTION S15AEF(X, IFAIL)
  INTEGER          IFAIL
  real            X
```

### 3 Description

Evaluates the error function,

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

For  $|x| \leq 2$ ,  $\operatorname{erf} x = x \sum_{r=0}' a_r T_r(t)$ , where  $t = \frac{1}{2}x^2 - 1$ .

For  $2 < |x| < x_{hi}$ ,  $\operatorname{erf} x = \operatorname{sign} x \left\{ 1 - \frac{e^{-x^2}}{|x|\sqrt{\pi}} \sum_{r=0}' b_r T_r(t) \right\}$ , where  $t = \frac{x-7}{x+3}$ .

For  $|x| \geq x_{hi}$ ,  $\operatorname{erf} x = \operatorname{sign} x$ .

$x_{hi}$  is the value above which  $\operatorname{erf} x = \pm 1$  within *machine precision*. Its value is given in the Users' Note for your implementation.

### 4 References

- [1] Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

### 5 Parameters

- 1: X — *real* *Input*  
*On entry:* the argument  $x$  of the function.
- 2: IFAIL — INTEGER *Input/Output*  
*On entry:* IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.  
*On exit:* IFAIL = 0 unless the routine detects an error (see Section 6).

### 6 Error Indicators and Warnings

Errors detected by the routine:

There are no failure exits from this routine. The parameter IFAIL has been included for consistency with other routines in this chapter.

## 7 Accuracy

On a machine with approximately 11 significant figures the routine agrees with available tables to 10 figures and consistency checking with S15ADF of the relation

$$\operatorname{erf} x + \operatorname{erfc} x = 1.0$$

shows errors in at worst the 11th figure.

## 8 Further Comments

None.

## 9 Example

The example program reads values of the argument  $x$  from a file, evaluates the function at each value of  $x$  and prints the results.

### 9.1 Program Text

**Note.** The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```

*      S15AEF Example Program Text
*      Mark 14 Revised.  NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER       (NIN=5,NOUT=6)
*      .. Local Scalars ..
      real            X, Y
      INTEGER          IFAIL
*      .. External Functions ..
      real            S15AEF
      EXTERNAL         S15AEF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S15AEF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '      X            Y            IFAIL'
      WRITE (NOUT,*)
20     READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S15AEF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y, IFAIL
      GO TO 20
40     STOP
*
99999  FORMAT (1X,1P,2e12.3,I7)
      END

```

## 9.2 Program Data

S15AEF Example Program Data

```
-6.0  
-4.5  
-1.0  
1.0  
4.5  
6.0
```

## 9.3 Program Results

S15AEF Example Program Results

X	Y	IFAIL
-6.000E+00	-1.000E+00	0
-4.500E+00	-1.000E+00	0
-1.000E+00	-8.427E-01	0
1.000E+00	8.427E-01	0
4.500E+00	1.000E+00	0
6.000E+00	1.000E+00	0

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