S17AKF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

S17AKF returns a value for the derivative of the Airy function $\mathrm{Bi}(x)$, via the routine name.

2 Specification

```
real FUNCTION S17AKF(X, IFAIL)
INTEGER              IFAIL
real                 X
```

3 Description

This routine calculates an approximate value for the derivative of the Airy function $\mathrm{Bi}(x)$. It is based on a number of Chebyshev expansions.

For $x < -5$,

$$\mathrm{Bi}'(x) = \sqrt[4]{-x}\left[-a(t)\sin z + \frac{b(t)}{\zeta}\cos z\right],$$

where $z = \frac{\pi}{4} + \zeta, \zeta = \frac{2}{3}\sqrt{-x^3}$ and $a(t)$ and $b(t)$ are expansions in the variable $t = -2\left(\frac{5}{x}\right)^3 - 1$.

For $-5 \leq x \leq 0$,

$$\mathrm{Bi}'(x) = \sqrt{3}(x^2 f(t) + g(t)),$$

where f and g are expansions in $t = -2\left(\frac{x}{5}\right)^3 - 1$.

For $0 < x < 4.5$,

$$\mathrm{Bi}'(x) = e^{3x/2} y(t),$$

where $y(t)$ is an expansion in $t = 4x/9 - 1$.

For $4.5 \leq x < 9$,

$$\mathrm{Bi}'(x) = e^{21x/8} u(t),$$

where $u(t)$ is an expansion in $t = 4x/9 - 3$.

For $x \geq 9$,

$$\mathrm{Bi}'(x) = \sqrt[4]{x} e^{z} v(t),$$

where $z = \frac{2}{3}\sqrt{x^3}$ and $v(t)$ is an expansion in $t = 2\left(\frac{18}{z}\right) - 1$.

For $|x| <$ the square of the ***machine precision***, the result is set directly to $\mathrm{Bi}'(0)$. This saves time and avoids possible underflows in calculation.

For large negative arguments, it becomes impossible to calculate a result for the oscillating function with any accuracy so the routine must fail. This occurs for $x < -\left(\frac{\sqrt{\pi}}{\epsilon}\right)^{4/7}$, where ϵ is the ***machine precision***.

For large positive arguments, where Bi' grows in an essentially exponential manner, there is a danger of overflow so the routine must fail.

4 References

[1] Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

5 Parameters

1: X — ***real*** *Input*

On entry: the argument x of the function.

2: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors detected by the routine:

IFAIL = 1

X is too large and positive. On soft failure the routine returns zero.

IFAIL = 2

X is too large and negative. On soft failure the routine returns zero.

7 Accuracy

For negative arguments the function is oscillatory and hence absolute error is appropriate. In the positive region the function has essentially exponential behaviour and hence relative error is needed. The absolute error, E, and the relative error ϵ, are related in principle to the relative error in the argument δ, by

$$E \simeq |x^2 \operatorname{Bi}(x)|\delta \quad \epsilon \simeq \left| \frac{x^2 \operatorname{Bi}(x)}{\operatorname{Bi}'(x)} \right| \delta.$$

In practice, approximate equality is the best that can be expected. When δ, ϵ or E is of the order of the ***machine precision***, the errors in the result will be somewhat larger.

For small x, positive or negative, errors are strongly attenuated by the function and hence will effectively be bounded by the ***machine precision***.

For moderate to large negative x, the error is, like the function, oscillatory. However, the amplitude of the absolute error grows like $\frac{|x|^{7/4}}{\sqrt{\pi}}$. Therefore it becomes impossible to calculate the function with any accuracy if $|x|^{7/4} > \frac{\sqrt{\pi}}{\delta}$.

For large positive x, the relative error amplification is considerable: $\frac{\epsilon}{\delta} \sim \sqrt{x^3}$. However, very large arguments are not possible due to the danger of overflow. Thus in practice the actual amplification that occurs is limited.

8 Further Comments

None.

9 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S17AKF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      real             S17AKF
      EXTERNAL         S17AKF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S17AKF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S17AKF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y, IFAIL
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,1P,2e12.3,I7)
      END
```

9.2 Program Data

```
S17AKF Example Program Data
             -10.0
             -1.0
             0.0
             1.0
             5.0
             10.0
             20.0
```

9.3 Program Results

```
S17AKF Example Program Results

     X           Y        IFAIL

  -1.000E+01   1.194E-01      0
  -1.000E+00   5.924E-01      0
   0.000E+00   4.483E-01      0
   1.000E+00   9.324E-01      0
   5.000E+00   1.436E+03      0
   1.000E+01   1.429E+09      0
   2.000E+01   9.382E+25      0
```