S17DHF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

S17DHF returns the value of the Airy function $\mathrm{Bi}(z)$ or its derivative $\mathrm{Bi}'(z)$ for complex z, with an option for exponential scaling.

2 Specification

```
SUBROUTINE S17DHF(DERIV, Z, SCALE, BI, IFAIL)
INTEGER           IFAIL
complex           Z, BI
CHARACTER*1       DERIV, SCALE
```

3 Description

This subroutine returns a value for the Airy function $\mathrm{Bi}(z)$ or its derivative $\mathrm{Bi}'(z)$, where z is complex, $-\pi < \arg z \leq \pi$. Optionally, the value is scaled by the factor $e^{|\,\mathrm{Re}(2z\sqrt{z}/3)|}$.

The routine is derived from the routine CBIRY in Amos [2]. It is based on the relations $\mathrm{Bi}(z) = \frac{\sqrt{z}}{\sqrt{3}}(I_{-1/3}(w) + I_{1/3}(w))$, and $\mathrm{Bi}'(z) = \frac{z}{\sqrt{3}}(I_{-2/3}(w) + I_{2/3}(w))$, where I_ν is the modified Bessel function and $w = 2z\sqrt{z}/3$.

For very large $|z|$, argument reduction will cause total loss of accuracy, and so no computation is performed. For slightly smaller $|z|$, the computation is performed but results are accurate to less than half of ***machine precision***. If Re z is too large, and the unscaled function is required, there is a risk of overflow and so no computation is performed. In all the above cases, a warning is given by the routine.

4 References

[1] Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

[2] Amos D E (1986) Algorithm 644: A portable package for Bessel functions of a complex argument and nonnegative order *ACM Trans. Math. Software* **12** 265–273

5 Parameters

1: DERIV — CHARACTER*1 *Input*

On entry: specifies whether the function or its derivative is required.

If DERIV = 'F' or 'f', $\mathrm{Bi}(z)$ is returned.

If DERIV = 'D' or 'd', $\mathrm{Bi}'(z)$ is returned.

Constraint: DERIV = 'F', 'f', 'D' or 'd'.

2: Z — ***complex*** *Input*

On entry: the argument z of the function.

3: SCALE — CHARACTER*1 *Input*

On entry: the scaling option.

If SCALE = 'U' or 'u', the result is returned unscaled.

If SCALE = 'S' or 's', the result is returned scaled by the factor $e^{|\operatorname{Re}(2z\sqrt{z}/3)|}$.

Constraint: SCALE = 'U', 'u', 'S' or 's'.

4: BI — ***complex*** *Output*

On exit: the required function or derivative value.

5: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, −1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or −1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors detected by the routine:

IFAIL = 1

On entry, DERIV ≠ 'F' or 'D'.
or SCALE ≠ 'U' or 'S'.

IFAIL = 2

No computation has been performed due to the likelihood of overflow, because real(Z) is too large – how large depends on the overflow threshold of the machine. This error exit can only occur when SCALE = 'U'.

IFAIL = 3

The computation has been performed, but the errors due to argument reduction in elementary functions make it likely that the result returned by S17DHF is accurate to less than half of ***machine precision***. This error exit may occur if ABS(Z) is greater than a machine-dependent threshold value (given in the Users' Note for your implementation).

IFAIL = 4

No computation has been performed because the errors due to argument reduction in elementary functions mean that all precision in the result returned by S17DHF would be lost. This error exit may occur if ABS(Z) is greater than a machine-dependent threshold value (given in the Users' Note for your implementation).

IFAIL = 5

No result is returned because the algorithm termination condition has not been met. This may occur because the parameters supplied to S17DHF would have caused overflow or underflow.

7 Accuracy

All constants in subroutine S17DHF are given to approximately 18 digits of precision. Calling the number of digits of precision in the floating-point arithmetic being used t, then clearly the maximum number of correct digits in the results obtained is limited by $p = \min(t, 18)$. Because of errors in argument reduction when computing elementary functions inside S17DHF, the actual number of correct digits is limited, in general, by $p - s$, where $s \approx \max(1, |\log_{10}|z||)$ represents the number of digits lost due to the argument reduction. Thus the larger the value of $|z|$, the less the precision in the result.

Empirical tests with modest values of z, checking relations between Airy functions $\mathrm{Ai}(z)$, $\mathrm{Ai}'(z)$, $\mathrm{Bi}(z)$ and $\mathrm{Bi}'(z)$, have shown errors limited to the least significant 3-4 digits of precision.

8 Further Comments

Note that if the function is required to operate on a real argument only, then it may be much cheaper to call S17AHF or S17AKF.

9 Example

The example program prints a caption and then proceeds to read sets of data from the input data stream. The first datum is a value for the parameter DERIV, the second is a complex value for the argument, Z, and the third is a value for the parameter SCALE. The program calls the routine and prints the results. The process is repeated until the end of the input data stream is encountered.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S17DHF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      complex          BI, Z
      INTEGER          IFAIL
      CHARACTER*1      DERIV, SCALE
*     .. External Subroutines ..
      EXTERNAL         S17DHF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S17DHF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*)
     +  'DERIV           Z         SCALE           BI         IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) DERIV, Z, SCALE
      IFAIL = 0
*
      CALL S17DHF(DERIV,Z,SCALE,BI,IFAIL)
*
      WRITE (NOUT,99999) DERIV, Z, SCALE, BI, IFAIL
      GO TO 20
   40 STOP
*
99999 FORMAT (3X,A,'   (',F8.4,',',F8.4,')   ',A,'   (',F8.4,',',F8.4,
     +       ')',I5)
      END
```

9.2 Program Data

```
S17DHF Example Program Data
 'F'   ( 0.3,  0.4)    'U'
 'F'   ( 0.2,  0.0)    'U'
 'F'   ( 1.1, -6.6)    'U'
 'F'   ( 1.1, -6.6)    'S'
 'D'   (-1.0,  0.0)    'U'
```

9.3 Program Results

```
S17DHF Example Program Results

DERIV           Z          SCALE           BI          IFAIL

  F   (  0.3000,  0.4000)   U   (  0.7355,  0.1825)    0
  F   (  0.2000,  0.0000)   U   (  0.7055,  0.0000)    0
  F   (  1.1000, -6.6000)   U   (-47.9039, 43.6634)    0
  F   (  1.1000, -6.6000)   S   ( -0.1300,  0.1185)    0
  D   ( -1.0000,  0.0000)   U   (  0.5924,  0.0000)    0
```