REPORT OF CALCULATION THE TORQUE OF THE GIANO FILTER AND SLIT WHEEL SYSTEM

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Table of contents

1. Introduction

The mechanical system of the GIANO slit and filter wheel is shown in Fig. 1. Each device consists of a worm gear which is driven by a stepper motor (VSS 32.200.1,2 UHVC made by Phytron).
The slit wheel consists 15 slit frames which are mounted inside stainless steel holders (slit mounts). These steel holders are elastically mounted on the Aluminium wheel of the slit wheel device. To achieve an accurate positioning of the slits in relation to the focal plane, a slit lock fork, also mounted on the mechanical system, presses the slit mount against three kinematic points (plane, cylinders and spheres). This three kinematic points are mounted on a rigid system behind the slit wheel.
Stepper motors which are driven with high currents can warm up the cold structure of the mechanical filter and slit wheel system. To reduce the heat introduction the acceleration torques must be reduced to a less amount. The acceleration torques to move the slit wheel, the filter wheel and also the slit lock fork are calculated in this report.

Figure 1 shows a schematical draw the mechanical system with all components:

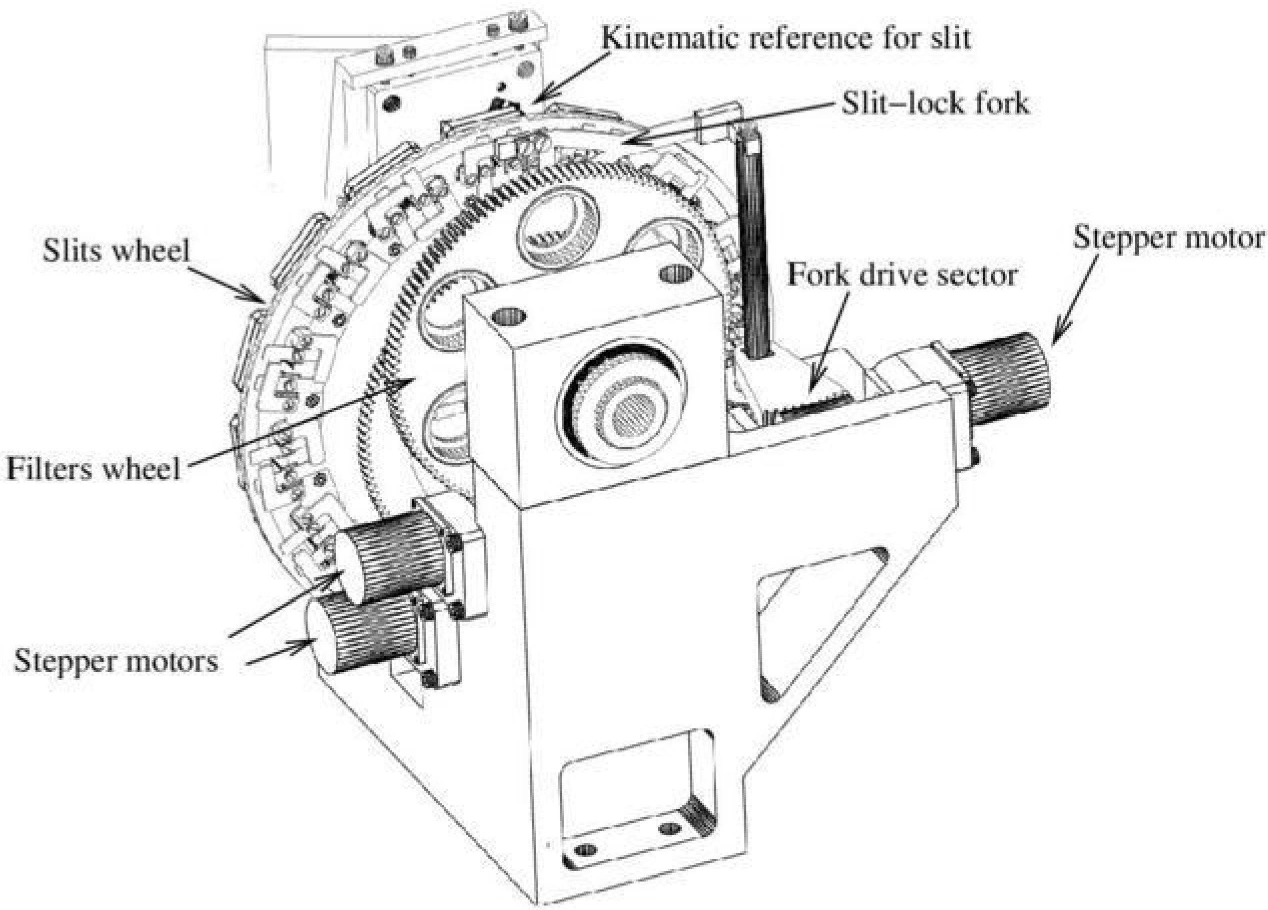

Fig. 1: Schematic draw of the Giano mechanical system holding the slit and the filter wheel.

2. Worm gear of the filter wheel

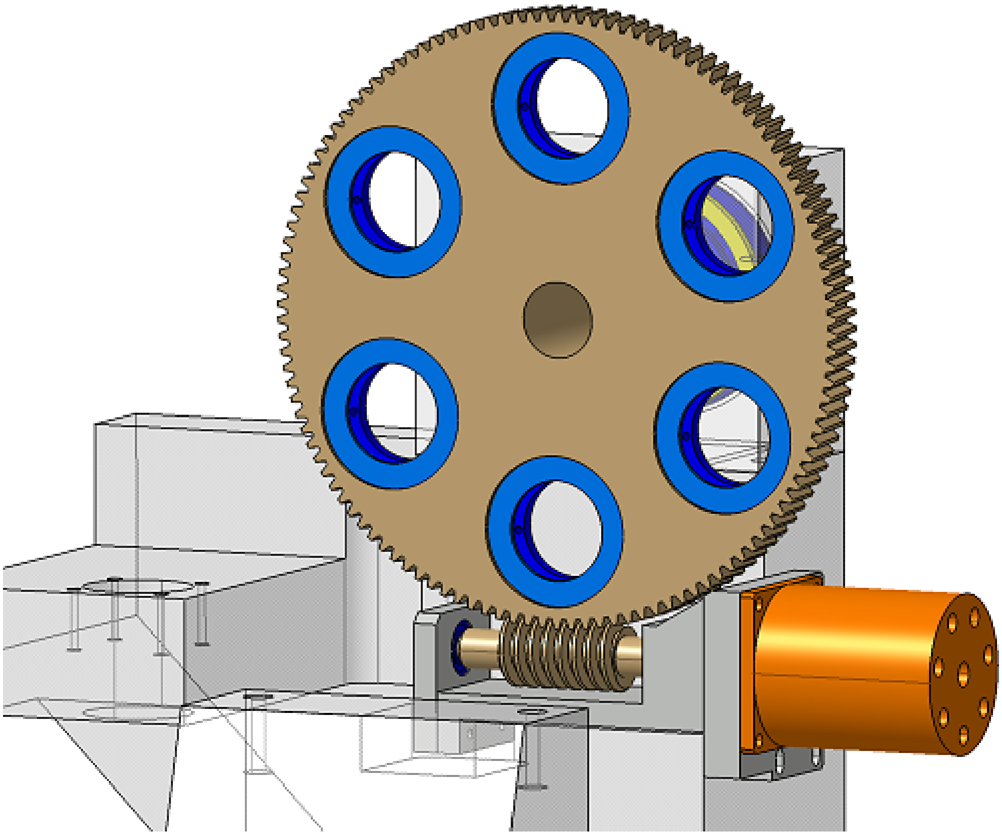

Fig. 2: Drawing of the filter wheel system mounted on the main structure

2.1 Gear main specifications

Gear ratio	i	1:	120
Moving frequency, related to full step mode	f	Hz	300
Acceleration time to achieve moving frequency	Δt	s	1

Gear ratio:

$$i=\frac{z2}{z1}=\frac{120}{1}=120$$

Rotational speed of worm and wheel in relation to the moving frequency:

$$n_{worm}=\frac{f}{200}=1,5/s=90/min$$ (standard step angle = 1,8°; 200 steps = 1 turn)

$$n_{wheel}=\frac{n_{worm}}{120}=1,25\exp(-02)/s=0,75/min$$ (1 turn = 90s)

Distance between the axes:

$$a=\frac{(dm1+dm2)}{2}=\frac{(12,043mm+120,416mm)}{2}=66,23mm$$

2.1.1 Worm specifications

Number of teeth	z1	-	1
Axial module	ma	mm	1,0035
Axial pitch	pa	mm	3,1525
Reference diameter	dm1	mm	12,043
Friction angle	ρ'	°	2
Pressure angle	α0	°	20
Lead angle	γm	°	4,7631
Volume	V	m^3	4,1exp(-06)
Weight	m	kg	3,22exp(-02)
Density of AISI 304 (X5CrNi18-10)	ρ	kg/m^3	7854

Catia P3 V5R10 was used to calculate the moment of inerta and the center of gravity. The results are transfered from the Catia calculation report:

Moment of inerta	J_{worm}	$kg \cdot m^2$	5,17exp(-07)
Position of gravity center	k	mm	0,259

Gravity force:

$$F_g = m \cdot g = 3{,}22\exp(-02)\,kg \cdot 9{,}81\,m/s^2 = 0{,}32\,N$$

Angular velocity:

$$\omega_{worm} = 2 \cdot \Pi \cdot n_{worm} = 2 \cdot \Pi \cdot 1{,}5/s = 9{,}42/s$$

Angular acceleration:

$$\alpha_{worm} = \frac{2 \cdot \Pi \cdot n_{worm}}{\Delta t} = \frac{2 \cdot \Pi \cdot 1{,}5/s}{1s} = 9{,}42\frac{rad}{s^2}$$

Acceleration torque:

$$T_a = J \cdot \frac{\Delta \omega}{\Delta t} = J \cdot \alpha \quad \text{(common)}$$

$$T_{a(worm)} = J_{worm} \cdot \alpha_{worm} = 5{,}17\exp(-07)\,kg \cdot m^2 \cdot 9{,}42\frac{rad}{s^2} = 4{,}87\exp(-06)Nm = \mathbf{4{,}87\exp(-03)mNm}$$

2.1.2 Wheel specifications

Number of teeth	z1	-	120
Normal module	mn	mm	1
Transverse module	ms	mm	1,0035
Reference diameter	dm2	mm	120,416
Reference circle diameter	d2	mm	120,416
Friction angle	ρ'	°	2
Pressure angle	α0	°	20
Lead angle	γm	°	4,7631
Weight	m	kg	0,773

Catia P3 V5R10 was used to calculate the moment of inerta and the center of gravity. The results are transfered from the Catia calculation report. The calculation to determine the moment of inerta of the filter wheel is done without consideration of weight of the six infrared filters:

Moment of inerta	J_{wheel}	$kg \cdot m^2$	1,094exp(-03)
Position of gravity center	-	mm	53,153

Gravity force:

$$F_g = m \cdot g = 0{,}773\,kg \cdot 9{,}81\,m/s^2 = 7{,}586\,N$$

Angular velocity:

$$\omega_{wheel} = 2 \cdot \Pi \cdot n_{wheel} = 2 \cdot \Pi \cdot 1{,}25\,\exp(-02)/s = 0{,}079/s$$

Angular acceleration:

$$\alpha_{wheel} = \frac{2 \cdot \Pi \cdot n_{wheel}}{\Delta t} = \frac{2 \cdot \Pi \cdot 1{,}25\,\exp(-02)/s}{1s} = 0{,}079\,\frac{rad}{s^2}$$

Acceleration torque:

$$T_{a(wheel)} = J_{wheel} \cdot \alpha_{wheel} = 1{,}094\,\exp(-03)\,kg \cdot m^2 \cdot 0{,}079\,\frac{rad}{s^2} = 8{,}6\,\exp(-05)\,Nm = \mathbf{8{,}6\,exp(-02)\,mNm}$$

Resulting force related to the wheel radius:

$$F_{res} = \frac{T_{a(wheel)}}{r_{wheel}} = \frac{8{,}6\,\exp(-02)\,mNm}{0{,}5 \cdot 120{,}416\,mm} = \mathbf{1{,}43\,exp(-03)\,N}$$

2.2 Bearings

2.2.1 Bearings of the worm

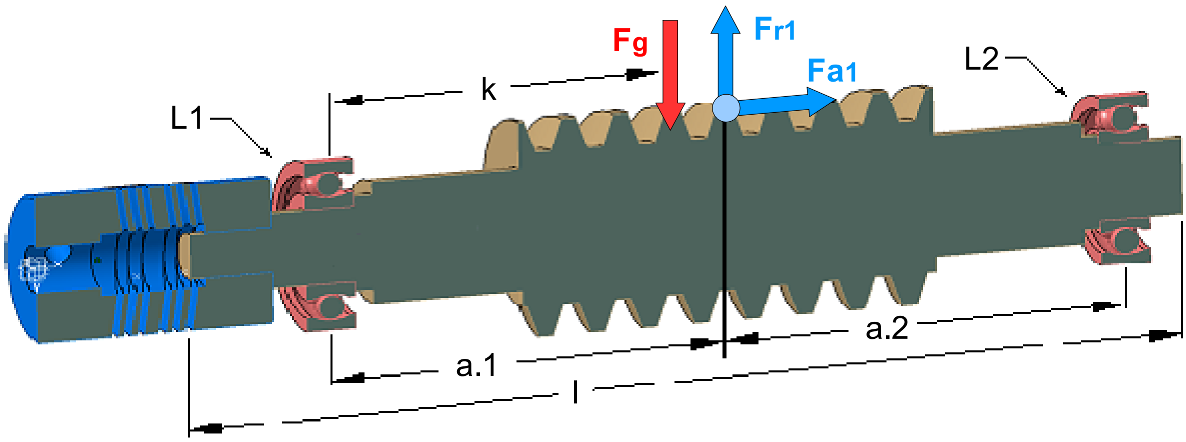

Fig. 3: Cross sectional view of the GIANO worm which contains the bearings (L1 + L2) and a flexible connecting element (Flexbeam-2)

Worm and bearing dimensions:

Distance	a.1	mm	25
Distance	a.2	mm	25
Lenght of the worm	l	mm	60
Inside diameter of bearing	d	mm	5

Friction coefficient of radial groved ball bearings	μ	-	0,0015

Position of center of gravity:

$$k = a.1 - k = 25mm - 0,259\,mm = 24,741\,mm$$

Acting forces to the bearings L1 and L2:

Splitting the gravity force to both bearings:

$$\sum F = 0 \quad F'_{L1} + F'_{L2} - F_g = 0$$

$$F'_{L2} = \frac{F_g \cdot k}{(a.1 + a.2)} = \frac{0,32\,N \cdot 24,741\,mm}{25mm + 25mm} = 0,156\,N$$

$$F'_{L1} = F_g - F'_{L2} = 0,32\,N - 0,156\,N = 0,160\,N$$

Equations to calculate the radial, tangential and axial force components:

Radial force:

$$F_{L1(radial)} = F_{r1} \cdot \left(\frac{a.1}{(a.1 + a.2)} \right) + F'_{L1} = 5{,}21\,\exp(-04)\,N \cdot \left(\frac{25mm}{(25mm + 25mm)} \right) + 0{,}160\,N = 0{,}160\,N$$

$$F_{L2(radial)} = F_{r1} \cdot \left(\frac{a.2}{(a.1 + a.2)} \right) + F'_{L2} = 5{,}21\,\exp(-04)\,N \cdot \left(\frac{25mm}{(25mm + 25mm)} \right) + 0{,}156\,N = 0{,}157\,N$$

Tangential force:

Distance a.1 is equal to a.2, therefore $F_{L1(tangential)} = F_{L2(tangential)}$

$$F_{L1(tangential)} = F_{L2(tangential)} = F_{t1} \cdot \left(\frac{a.2}{(a.1 + a.2)} \right) = 1{,}69\,\exp(-04)\,N \cdot \left(\frac{25mm}{(25mm + 25mm)} \right) = 8{,}5\,\exp(-05)\,N$$

Axial force:

Distance a.1 is equal to a.2, therefore $F_{L1(axial)} = F_{L2(axial)}$

$$F_{L1(axial)} = F_{L2(axial)} = F_{a1} \cdot \left(\frac{dm1}{2 \cdot (a.1 + a.2)} \right) = 1{,}43\,\exp(-03)\,N \cdot \left(\frac{12{,}043\,mm}{2 \cdot (25mm + 25mm)} \right) = 1{,}72\,\exp(-04)\,N$$

Resulting forces of the bearings L1 and L2:

$$F_{L1} = \sqrt{(F_{L1(radial)} + F_{L1(axial)})^2 + F^2_{L1(tangential)}} = \sqrt{(0{,}160\,N + 1{,}72\,\exp(-04)\,N)^2 + 8{,}5\,\exp(-05)\,N^2} = 0{,}16\,N$$

$$F_{L2} = \sqrt{(F_{L2(radial)} - F_{L2(axial)})^2 + F^2_{L2(tangential)}} = \sqrt{(0{,}157\,N - 1{,}72\,\exp(-04)\,N)^2 + 8{,}5\,\exp(-05)\,N^2} = 0{,}16\,N$$

Approximated calculation of the friction torque without considering of the lubricant:

$$M_{friction(L1)} = F_{L1} \cdot \mu \cdot \frac{d}{2} = 0{,}16\,N \cdot 0{,}0015 \cdot \frac{5mm}{2} = 6{,}0\,\exp(-04)\,mNm$$

$$M_{friction(L2)} = F_{L2} \cdot \mu \cdot \frac{d}{2} = 0{,}26\,N \cdot 0{,}0015 \cdot \frac{5mm}{2} = 5{,}87\,\exp(-04)\,mNm$$

$$M_{friction(L1+L2)} = M_{friction(L1)} + M_{friction(L2)} = \mathbf{1{,}2\,\exp(-03)\,mNm}$$

2.2.2 Bearings of the wheel

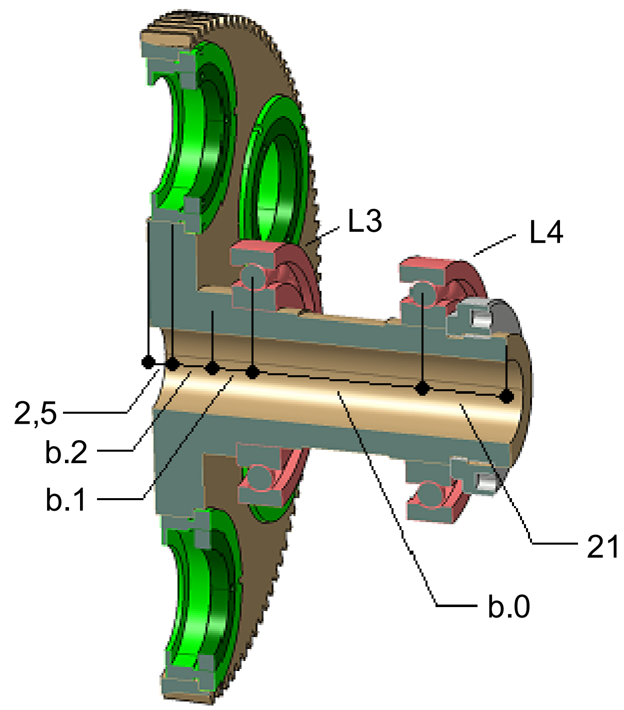

Fig. 4: Dimensions of the wheel to calculate the acting forces.

Distance	b.0	mm	31
Distance	b.1	mm	1,15
Distance	b.2	mm	7,35
Lenght of the axis	l	mm	65
Inside diameter of bearing	d	mm	25

Acting forces to the bearings L3 and L4:

Radial force:

$$\sum F_{radial}=0; \quad -F_{r2}+F_G-F_{L3(radial)}+F_{L4(radial)}=0$$

$$F_{L3(radial)}=\frac{F_g\cdot(b.0+b.1)-F_{r2}\cdot(b.0+b.1+b.2)+F_{a2}\cdot\frac{dm2}{2}}{b.0}$$

$$F_{L3(radial)}=\frac{7,586\,N\cdot 32,15\,mm-5,21\exp(-04)N\cdot 39,5\,mm+1,69\exp(-04)N\cdot 60,208\,mm}{31\,mm}=7,87\,N$$

$$F_{L4(radial)}=-F_g+F_{L3(radial)}+F_{r2}=-7,586\,N+7,87\,N+5,21\exp(-04)N=0,28\,N$$

Axial force:

$$F_{L3(axial)}=F_{t1}=1,69\exp(-04)N$$

→ Bearing L4 is not a fixed bearing, therefore the axial force is 0 N.

Resulting forces of the bearings L3 and L4:

$$F_{L3}=\sqrt{F^2_{L3(axial)}+F^2_{L3(radial)}}=\sqrt{1{,}69\exp(-04)N^2+7{,}87\,N^2}=7{,}868\,N$$

$$F_{L4}=\sqrt{F^2_{L4(axial)}+F^2_{L4(radial)}}=\sqrt{0^2+0{,}28\,N^2}=0{,}28\,N$$

Approximated calculation of the friction torque without considering of the lubricant:

$$M_{friction(L3)}=F_{L3}\cdot\mu\cdot\frac{d}{2}=7{,}868\,N\cdot 0{,}0015\cdot\frac{25mm}{2}=0{,}15\,mNm$$

$$M_{friction(L4)}=F_{L4}\cdot\mu\cdot\frac{d}{2}=0{,}28\,N\cdot 0{,}0015\cdot\frac{25mm}{2}=5{,}29\exp(-03)mNm$$

$$M_{friction(L3+L4)}=M_{friction(L3)}+M_{friction(L4)}=0{,}150\,mNm$$

2.3 Torques and Forces of the worm gear

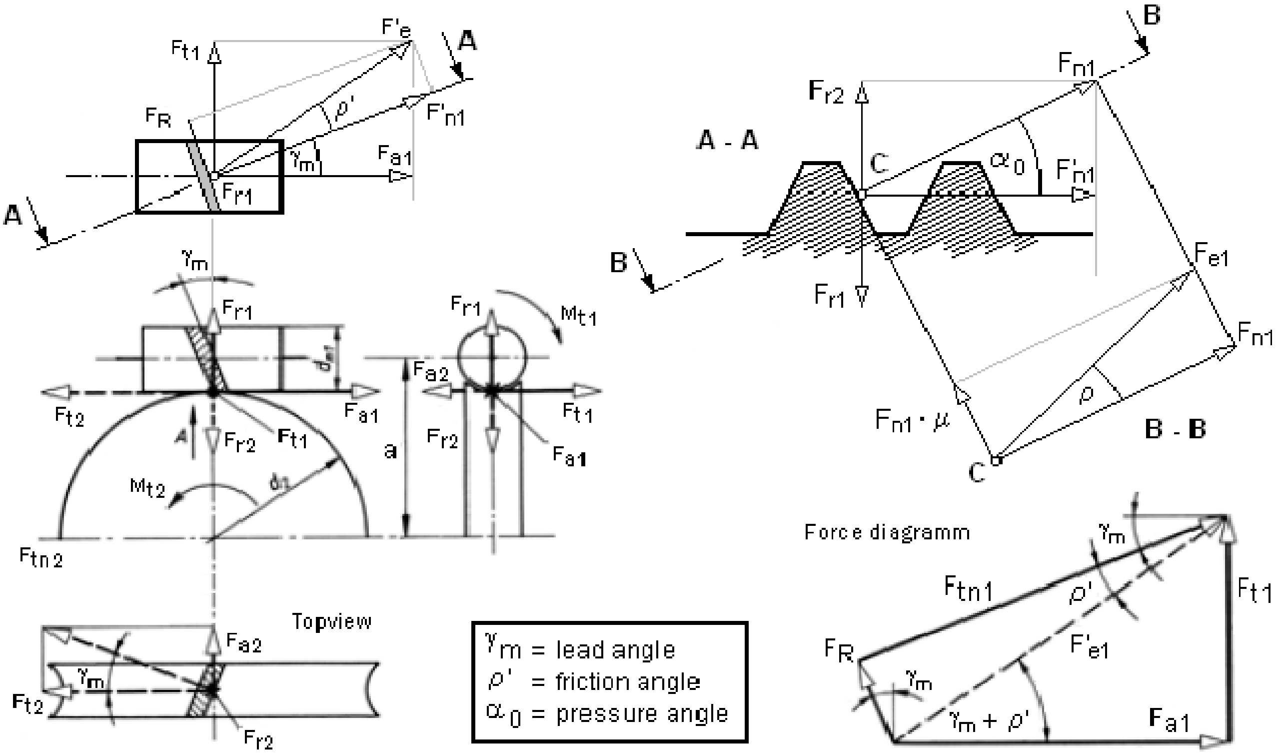

Fig. 5: Acting forces on a worm gear, when worm is the driven part.

Acting forces at the contact point between the worm and the wheel:

On the roll point of a worm gear interacts two contact forces between worm and wheel. Whereas both interacts to each other contrarily because of *Actio* = *Reactio*. The three components are:

F_t	=	tangential force
F_r	=	radial force
F_a	=	axial force

Because of *Actio* = *Reactio* the following forces are identical:

$$F_{t1}=F_{a2} \qquad F_{r1}=F_{r2} \qquad F_{a1}=F_{t2}$$

Resulting force of in relation of the wheel radius:

F_{res} is the basis force to calculate the further forces which are acting on the roll point of the worm gear and is equal to the forces F_{t2} and F_{a1}.

$$F_{res}=F_{t2}=F_{a1}=1{,}43\,\exp(-03)\,N$$

Tangential force F_{t1}:

$$F_{t1}=F_{a1}\cdot\tan(\gamma m+\rho')=1{,}43\exp(-03)\,N\cdot\tan(4{,}7631+2)=1{,}69\exp(-04)\,N$$

Backup force F'_e:

$$F'_{e1} = \sqrt{F_{a1}^2 + F_{t1}^2} = \sqrt{1{,}43\exp(-03)N^2 + 1{,}69\exp(-04)N^2} = 1{,}437\exp(-03)N$$

Projection of normal force F_{n1}:

$$F'_{n1} = F'_{e1} \cdot \cos(\gamma m) = 1{,}437\exp(-03)N \cdot \cos(4{,}7631) = 1{,}432\exp(-03)N$$

Normal force F_{n1}:

$$F_{n1} = \frac{F'_{n1}}{\cos(\alpha 0)} = \frac{1{,}432\exp(-03)N}{\cos(20)} = 1{,}524\exp(-03)N$$

Radial force F_{r1}:

$$F_{r1} = F_{n1} \cdot \sin(\alpha 0) = 1{,}524\exp(-03)N \cdot \sin(20) = 5{,}21\exp(-04)N$$

Friction force F_R:

Friction coefficient of stainless steel and bronze: $\mu = 0{,}180$

$$F_R = F_{n1} \cdot \mu = 1{,}524\exp(-03)N \cdot 0{,}180 = 2{,}74\exp(-04)N$$

Component of the friction force which is perpendicular to the worm axis F'_R:

$$F'_R = F_R \cdot \cos(\gamma m) = 2{,}74\exp(-04)N \cdot \cos(4{,}7631) = 2{,}73\exp(-04)N$$

Static friction torque T_{sf} of the worm:

$$T_{sf} = F'_R \cdot \frac{1}{2} \cdot dm1 = 2{,}73\exp(-04)N \cdot \frac{1}{2} \cdot 12{,}043mm = \mathbf{1{,}65\exp(-03)mNm}$$

Moment of inerta of the stepper motor and the Flexbeam-2:

Phytron stepper motor VSS 32.200.1,2	J	$kg \cdot m^2$	1,0exp(-06)
Flexbeam-2 / Type: PSMR10	J	$kg \cdot m^2$	2,9exp(-08)

Acceleration torque of the stepper motor:

$$T_{a(rotor)} = J_{worm} \cdot \alpha_{worm} = 1{,}0\exp(-06)kg \cdot m^2 \cdot 9{,}42\frac{rad}{s^2} = 9{,}42\exp(-06)Nm = \mathbf{9{,}42\exp(-03)mNm}$$

Acceleration torque of the Flexbeam-2:

$$T_{a(flexbam-2)} = J_{worm} \cdot \alpha_{worm} = 2{,}9\exp(-08)kg \cdot m^2 \cdot 9{,}42\frac{rad}{s^2} = 2{,}73\exp(-07)Nm = \mathbf{2{,}73\exp(-04)mNm}$$

2.4 Efficiency of the worm gear

Efficiency of a radial grooved ball bearing	η_w	-	0,96

(Capacity flux from the worm to the wheel)

$$\eta_{12}=\frac{P_1}{P_2}=\eta_w^2\cdot\frac{\tan\gamma_m}{\tan(\gamma_m+\rho)}=0{,}96^2\cdot\frac{\tan(4{,}7631)}{\tan(4{,}7631+2)}=0{,}65$$

2.5 Results

$$T_{start}=T'_{a(wheel)}+T_{sf}+T_{a(worm)}+T_{a(rotor)}+M_{friction(L1+L2)}+T_{a(flexbeam-2)}$$

$$T'_{a(wheel)}=\frac{T_{a(wheel)}+M_{friction(L3+L4)}}{i\cdot\eta_{12}}=\frac{8{,}6\exp(-02)\,mNm+0{,}150\,mNm}{120\cdot 0{,}65}=\mathbf{3{,}06\exp(-03)\,mNm}$$

The value of the calculated starting torque, as shown in the following result, is required to accelerate the worm gear of the filter wheel. This result is in relation to the moving frequency f and and the accelerating time Δt of the ramp. Moreover this calculations includes also the acceleration torque of the rotor. This result is calculated for ambient temperature:

$$T_{start}=\mathbf{2{,}05\exp(-02)\,mNm}$$

3. Worm gear of the slit wheel

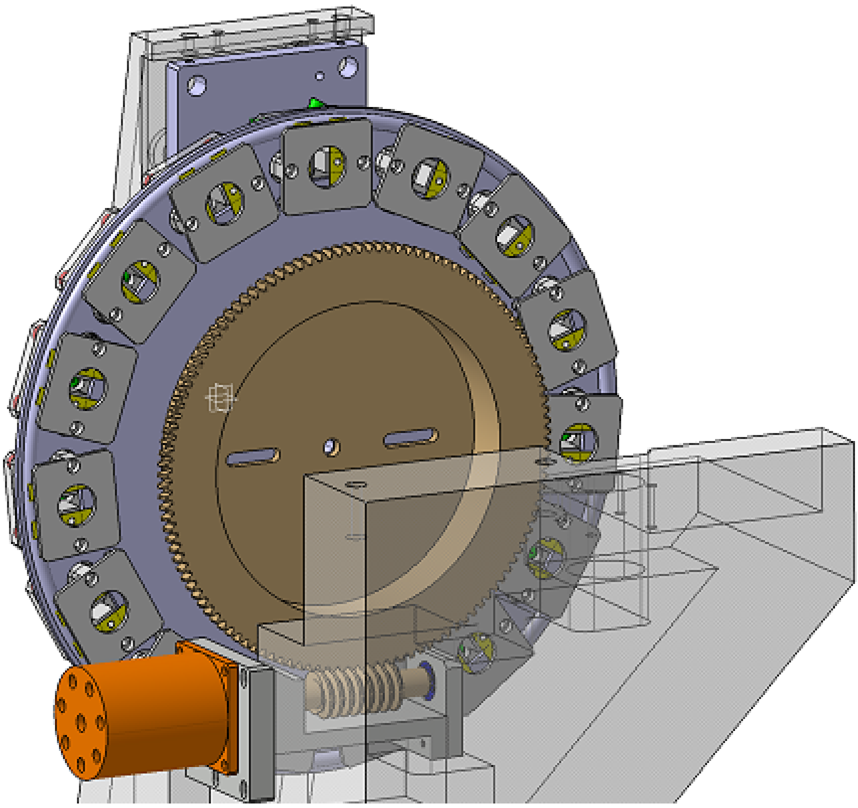

Fig. 6: Drawing of the slit wheel system mounted on the main structure

3.1 Gear main specifications

Gear ratio	i	1:	120
Moving frequency, related to full step mode	f	Hz	100
Acceleration time to achieve moving frequency	Δt	s	1

Gear ratio:

$$i=\frac{z2}{z1}=\frac{120}{1}=120$$

Rotational speed of worm and wheel in relation to the moving frequency:

$$n_{worm}=\frac{f}{200}=0,5/s=30/min$$ (standard step angle = 1,8°; 200 steps = 1 turn)

$$n_{wheel}=\frac{n_{worm}}{120}=4,17\exp(-03)/s=0,25/min$$ (1 turn = 4min)

Distance between the axes:

$$a=\frac{(dm1+dm2)}{2}=\frac{(12,043mm+120,416mm)}{2}=66,23mm$$

3.1.1 Worm specifications

Number of teeth	z1	-	1
Axial module	ma	mm	1,0035
Axial pitch	pa	mm	3,1525
Reference diameter	dm1	mm	12,043
Friction angle	ρ'	°	2
Pressure angle	α0	°	20
Lead angle	γm	°	4,7631
Volume	V	m^3	4,1exp(-06)
Weight	m	kg	3,22exp(-02)
Density of AISI 304 (X5CrNi18-10)	ρ	kg/m^3	7854

Catia P3 V5R10 was used to calculate the moment of inerta and the center of gravity. The results are transfered from the Catia calculation report:

Moment of inerta	J_{worm}	$kg{\cdot}m^2$	5,17exp(-07)
Position of gravity center	k	mm	0,259

Gravity force:

$$F_g = m \cdot g = 3{,}22\exp(-02)\,kg \cdot 9{,}81\,m/s^2 = 0{,}32\,N$$

Angular velocity:

$$\omega_{worm} = 2 \cdot \Pi \cdot n_{worm} = 2 \cdot \Pi \cdot 0{,}5/s = 3.14/s$$

Angular acceleration:

$$\alpha_{worm} = \frac{2 \cdot \Pi \cdot n_{worm}}{\Delta t} = \frac{2 \cdot \Pi \cdot 0{,}5/s}{1s} = 3{,}14\,\frac{rad}{s^2}$$

Acceleration torque:

$$T_{a(worm)} = J_{worm} \cdot \alpha_{worm} = 5{,}17\exp(-07)\,kg \cdot m^2 \cdot 3{,}14\,\frac{rad}{s^2} = 1{,}62\exp(-06)\,Nm = \mathbf{1{,}62\exp(-03)\,mNm}$$

3.1.2 Wheel specifications

Number of teeth	z1	-	120
Normal module	mn	mm	1
Transverse module	ms	mm	1,0035
Reference diameter	dm2	mm	120,416
Reference circle diameter	d2	mm	120,416
Friction angle	ρ'	°	2
Pressure angle	α0	°	20
Lead angle	γm	°	4,7631
Weight	m	kg	1,799

Catia P3 V5R10 was used to calculate the moment of inerta and the center of gravity. The results are transfered from the Catia calculation report. The calculation to determine the moment of inerta of the slit wheel is done with consideration of weight of the 15 slits:

Moment of inerta	J_{wheel}	$kg \cdot m^2$	7exp(-03)
Position of gravity center	-	mm	59,876

Gravity force:

$$F_g = m \cdot g = 1{,}799\,kg \cdot 9{,}81\,m/s^2 = 17{,}65\,N$$

Angular velocity:

$$\omega_{wheel} = 2 \cdot \Pi \cdot n_{wheel} = 2 \cdot \Pi \cdot 4{,}17\,exp(-03)/s = 0{,}026/s$$

Angular acceleration:

$$\alpha_{wheel} = \frac{2 \cdot \Pi \cdot n_{wheel}}{\Delta t} = \frac{2 \cdot \Pi \cdot 4{,}17\,exp(-03)/s}{1s} = 0{,}026\,\frac{rad}{s^2}$$

Acceleration torque:

$$T_{a(wheel)} = J_{wheel} \cdot \alpha_{wheel} = 7{,}0\,exp(-03)\,kg \cdot m^2 \cdot 0{,}026\,\frac{rad}{s^2} = 1{,}83\,exp(-04)\,Nm = \mathbf{0{,}183\,mNm}$$

Resulting force related to the wheel radius:

$$F_{res} = \frac{T_{a(wheel)}}{r_{wheel}} = \frac{0{,}183\,mNm}{0{,}5 \cdot 120{,}416\,mm} = \mathbf{3{,}05\,exp(-03)\,N}$$

3.2 Bearings

3.2.1 Bearings of the worm

The dimensions of the worm and the equations to divide the gravity force to the bearings L1 and L2 are equal to point 2.2.1.

Equations to calculate the radial, tangential and axial force components:

Radial force:

$$F_{L1(radial)} = F_{r1} \cdot \left(\frac{a.1}{(a.1 + a.2)} \right) + F'_{L1} = 1{,}11\,\exp(-03)N \cdot \left(\frac{25mm}{(25mm + 25mm)} \right) + 0{,}160\,N = 0{,}160\,N$$

$$F_{L2(radial)} = F_{r1} \cdot \left(\frac{a.2}{(a.1 + a.2)} \right) + F'_{L2} = 1{,}11\,\exp(-03)N \cdot \left(\frac{25mm}{(25mm + 25mm)} \right) + 0{,}156\,N = 0{,}157\,N$$

Tangential force:

Distance a.1 is equal to a.2, therefore $F_{L1(tangential)} = F_{L2(tangential)}$

$$F_{L1(tangential)} = F_{L2(tangential)} = F_{t1} \cdot \left(\frac{a.2}{(a.1 + a.2)} \right) = 3{,}61\,\exp(-04)\,N \cdot \left(\frac{25mm}{(25mm + 25mm)} \right) = 1{,}8\,\exp(-04)\,N$$

Axial force:

Distance a.1 is equal to a.2, therefore $F_{L1(axial)} = F_{L2(axial)}$

$$F_{L1(axial)} = F_{L2(axial)} = F_{a1} \cdot \left(\frac{dm1}{2 \cdot (a.1 + a.2)} \right) = 3{,}04\,\exp(-03)\,N \cdot \left(\frac{12{,}043\,mm}{2 \cdot (25mm + 25mm)} \right) = 3{,}67\,\exp(-04)\,N$$

Resulting forces of the bearings L1 and L2:

$$F_{L1} = \sqrt{(F_{L1(radial)} + F_{L1(axial)})^2 + F^2_{L1(tangential)}} = \sqrt{(0{,}160\,N + 3{,}67\,\exp(-04)\,N)^2 + 1{,}8\,\exp(-04)N^2} = 0{,}161\,N$$

$$F_{L2} = \sqrt{(F_{L2(radial)} - F_{L2(axial)})^2 + F^2_{L2(tangential)}} = \sqrt{(0{,}157\,N - 3{,}67\,\exp(-04)\,N)^2 + 1{,}8\,\exp(-04)N^2} = 0{,}157\,N$$

Approximated calculation of the friction torque without considering of the lubricant:

$$M_{friction(L1)} = F_{L1} \cdot \mu \cdot \frac{d}{2} = 0{,}161\,N \cdot 0{,}0015 \cdot \frac{5mm}{2} = 6{,}02\,\exp(-04)\,mNm$$

$$M_{friction(L2)} = F_{L2} \cdot \mu \cdot \frac{d}{2} = 0{,}157\,N \cdot 0{,}0015 \cdot \frac{5mm}{2} = 5{,}87\,\exp(-04)\,mNm$$

$$M_{friction(L1+L2)} = M_{friction(L1)} + M_{friction(L2)} = \mathbf{1{,}2\,\exp(-03)\,mNm}$$

3.2.2 Bearings of the wheel

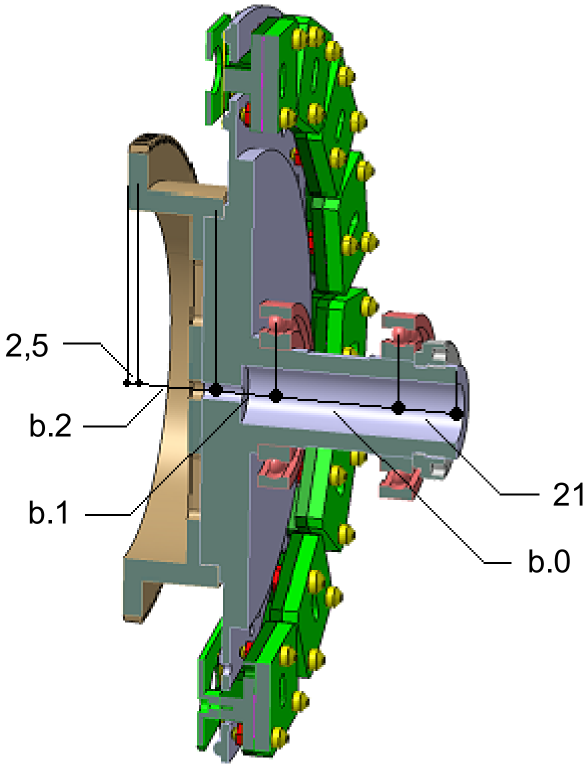

Fig. 7: Dimensions of the wheel to calculate the acting forces.

Distance	b.0	mm	31
Distance	b.1	mm	7,88
Distance	b.2	mm	21,12
Lenght of the axis	l	mm	83,5
Inside diameter of bearing	d	mm	25

Acting forces to the bearings L3 and L4:

Radial force:

$$\sum F_{radial}=0; \quad -F_{r2}+F_g-F_{L3(radial)}+F_{L4(radial)}=0$$

$$F_{L3(radial)}=\frac{F_g\cdot(b.0+b.1)-F_{r2}\cdot(b.0+b.1+b.2)+F_{a2}\cdot\frac{dm2}{2}}{b.0}$$

$$F_{L3(radial)}=\frac{17{,}648N\cdot 38{,}88\,mm-1{,}11\exp(-03)N\cdot 60mm+3{,}61\exp(-04)N\cdot 60{,}208mm}{31mm}=22{,}13N$$

$$F_{L4(radial)}=-F_g+F_{L3(radial)}+F_{r2}=-17{,}648N+22{,}13N+1{,}11\exp(-03)N=4{,}48N$$

Axial force:

$$F_{L3(axial)} = F_{t1} = 3{,}61\exp(-04)\,N$$

→ Bearing L4 is not a fixed bearing, therefore the axial force is 0 N.

Resulting forces of the bearings L3 and L4:

$$F_{L3} = \sqrt{F_{L3(axial)}^2 + F_{L3(radial)}^2} = \sqrt{3{,}61\exp(-04)N^2 + 22{,}13N^2} = 22{,}13\,N$$

$$F_{L4} = \sqrt{F_{L4(axial)}^2 + F_{L4(radial)}^2} = \sqrt{0^2 + 4{,}48N^2} = 4{,}48\,N$$

Approximated calculation of the friction torque without considering of the lubricant:

$$M_{friction(L3)} = F_{L3} \cdot \mu \cdot \frac{d}{2} = 22{,}13N \cdot 0{,}0015 \cdot \frac{25mm}{2} = 0{,}415\,mNm$$

$$M_{friction(L4)} = F_{L4} \cdot \mu \cdot \frac{d}{2} = 4{,}48N \cdot 0{,}0015 \cdot \frac{25mm}{2} = 8{,}4\exp(-02)\,mNm$$

$$M_{friction(L3+L4)} = M_{friction(L3)} + M_{friction(L4)} = 0{,}499\,mNm$$

3.3 Torques, Forces and Efficiency of the worm gear

Resulting force of in relation of the wheel radius:

F_{res} is the basis force to calculate the further forces which are acting on the roll point of the worm gear. F_{res} is equal to F_{t2} and F_{a1}.

$$F_{res}=F_{t2}=F_{a1}=3{,}05\,\exp(-03)\,N$$

Tangential force F_{t1}:

$$F_{t1}=F_{a1}\cdot\tan(\gamma m+\rho')=3{,}05\exp(-03)N\cdot\tan(4{,}7631+2)=3{,}62\exp(-04)N$$

Backup force F'_{e}:

$$F'_{e1}=\sqrt{F_{a1}^2+F_{t1}^2}=\sqrt{3{,}05\exp(-03)N^2+3{,}62\exp(-04)N^2}=3{,}07\exp(-03)N$$

Projection of normal force F_{n1}:

$$F'_{n1}=F'_{e1}\cdot\cos(\gamma m)=3{,}07\exp(-03)N\cdot\cos(4{,}7631)=3{,}06\exp(-03)N$$

Normal force F_{n1}:

$$F_{n1}=\frac{F'_{n1}}{\cos(\alpha 0)}=\frac{3{,}06\exp(-03)N}{\cos(20)}=3{,}25\exp(-03)N$$

Radial force F_{r1}:

$$F_{r1}=F_{n1}\cdot\sin(\alpha 0)=3{,}25\exp(-03)N\cdot\sin(20)=1{,}11\exp(-03)N$$

Friction force F_{R}:

Friction coefficient of stainless steel and bronze: $\mu=0{,}180$

$$F_{R}=F_{n1}\cdot\mu=3{,}25\exp(-03)N\cdot 0{,}180=5{,}85\exp(-04)N$$

Component of the friction force which is perpendicular to the worm axis F'_{R}:

$$F'_{R}=F_{R}\cdot\cos(\gamma m)=5{,}85\exp(-04)N\cdot\cos(4{,}7631)=5{,}83\exp(-04)N$$

Static friction moment T_{sf} of the worm:

$$T_{sf}=F'_{R}\cdot\frac{1}{2}\cdot dm1=5{,}83\exp(-04)N\cdot\frac{1}{2}\cdot 12{,}043mm=\mathbf{3{,}51\exp(-03)mNm}$$

Moment of inerta of the stepper motor and the Flexbeam-2:

Phytron stepper motor VSS 32.200.1,2	J	kg·m^2	1,0exp(-06)
Flexbeam-2 / Type: PSMR10	J	kg·m^2	2,9exp(-08)

Accelerations torque of the stepper motor:

$$T_{a(rotor)} = J_{worm} \cdot \alpha_{worm} = 1{,}0\exp(-06)\,kg \cdot m^2 \cdot 3{,}14\,\frac{rad}{s^2} = 3{,}14\exp(-06)Nm = \mathbf{3{,}14\exp(-03)mNm}$$

Accelerations torque of the Flexbeam-2:

$$T_{a(flexbam-2)} = J_{worm} \cdot \alpha_{worm} = 2{,}9\exp(-08)\,kg \cdot m^2 \cdot 3{,}14\,\frac{rad}{s^2} = 9{,}11\exp(-08)Nm = \mathbf{9{,}11\exp(-05)mNm}$$

3.4 Efficiency of the worm gear

The Efficiency of the slit wheel worm gear is equal to point 2.4 in this report.

3.5 Conclusion

$$T_{start} = T'_{a(wheel)} + T_{sf} + T_{a(worm)} + T_{a(rotor)} + M_{friction\,(L1+L2)} + T_{a(flexbeam-2)}$$

$$T'_{a(wheel)} = \frac{T_{a(wheel)} + M_{friction(L3+L4)}}{i \cdot \eta_{12}} = \frac{0{,}183\,mNm + 0{,}499\,mNm}{120 \cdot 0{,}65} = \mathbf{8{,}78\exp(-03)mNm}$$

The value of the calculated starting torque, as shown in the following result, is required to accelerate the worm gear of the slit wheel:

$$T_{start} = \mathbf{1{,}84\exp(-02)mNm}$$

4. Worm gear of the slit lock fork

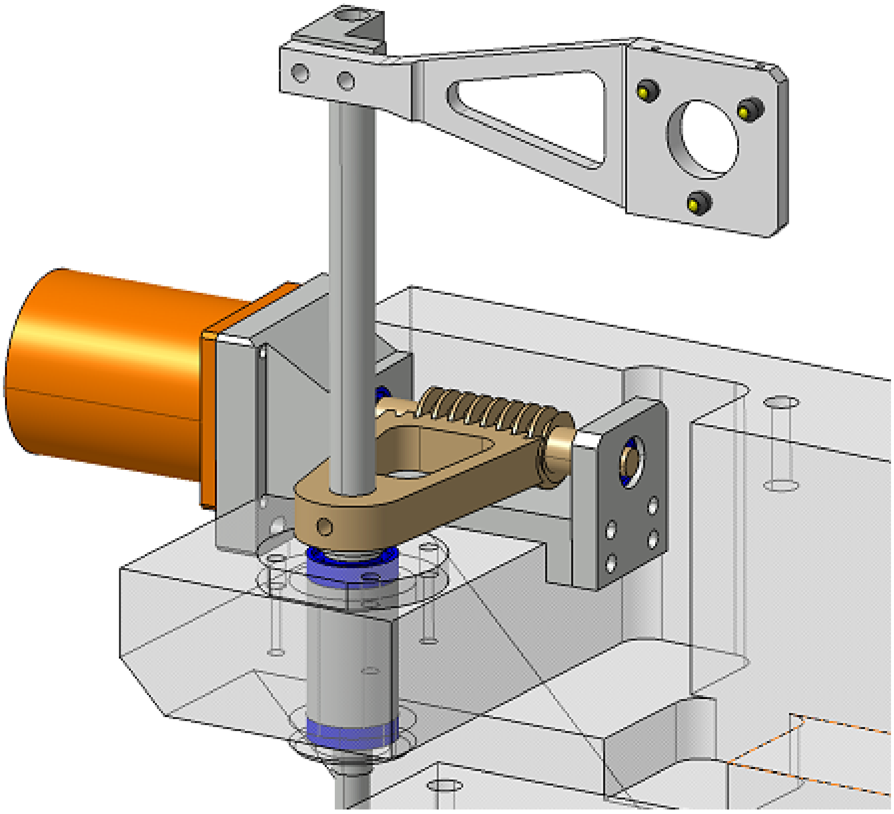

Fig. 8: Drawing of the slit lock fork system mounted on the main structure

4.1 Gear main specifications

Gear ratio	i	1:	80
Moving frequency, related to full step mode	f	Hz	300
Acceleration time to achieve moving frequency	Δt	s	1

Gear ratio:

$$i=\frac{z2}{z1}=\frac{80}{1}=80$$

Rotational speed of worm and wheel in relation to the moving frequency:

$$n_{worm}=\frac{f}{200}=1{,}5/s=90/\min$$ (standard step angle = 1,8°; 200 steps = 1 turn)

$$n_{wheel}=\frac{n_{worm}}{80}=1{,}875\exp(-02)/s=1{,}125/\min$$ (1 turn = 53,33s)

Distance between the axes:

$$a=\frac{(dm1+dm2)}{2}=\frac{(12{,}043\,mm+80{,}277\,mm)}{2}=46{,}16\,mm$$

4.1.1 Worm specifications

Number of teeth	z1	-	1
Axial module	ma	mm	1,0035
Axial pitch	pa	mm	3,1525
Reference diameter	dm1	mm	12,043
Friction angle	ρ'	°	2
Pressure angle	α0	°	20
Lead angle	γm	°	4,7631
Volume	V	m^3	4,1exp(-06)
Weight	m	kg	3,22exp(-02)
Density of AISI 304 (X5CrNi18-10)	ρ	kg/m^3	7854

Catia P3 V5R10 was used to calculate the moment of inerta and the center of gravity. The results are transfered from the Catia calculation report:

Moment of inerta	J_{worm}	$kg \cdot m^2$	5,17exp(-07)
Position of gravity center	k	mm	0,259

Gravity force:

$$F_g = m \cdot g = 3{,}22 \exp(-02)\, kg \cdot 9{,}81\, m/s^2 = 0{,}32\, N$$

Angular velocity:

$$\omega_{worm} = 2 \cdot \Pi \cdot n_{worm} = 2 \cdot \Pi \cdot 1{,}5/s = 9{,}42/s$$

Angular acceleration:

$$\alpha_{worm} = \frac{2 \cdot \Pi \cdot n_{worm}}{\Delta t} = \frac{2 \cdot \Pi \cdot 1{,}5/s}{1s} = 9{,}42 \frac{rad}{s^2}$$

Acceleration torque:

$$T_{a(worm)} = J_{worm} \cdot \alpha_{worm} = 5{,}17 \exp(-07)\, kg \cdot m^2 \cdot 9{,}42 \frac{rad}{s^2} = 4{,}87 \exp(-06) Nm = \mathbf{4{,}87 \exp(-03)\, mNm}$$

4.1.2 Wheel specifications

Number of teeth	z1	-	80
Axial module	ma	mm	1,0035
Reference diameter	dm2	mm	80,277
Reference circle diameter	d2	mm	80,277
Friction angle	ρ'	°	2
Pressure angle	α0	°	20
Lead angle	γm	°	4,7631
Weight	m	kg	0,134

Catia P3 V5R10 was used to calculate the moment of inerta and the center of gravity. The results are transfered from the Catia calculation report:

Moment of inerta	J_{wheel}	$kg \cdot m^2$	3,0exp(-04)
Position of gravity center	X	mm	3,49
Position of gravity center	Y	mm	6,88

Gravity force:

$$F_g = m \cdot g = 0{,}134\,kg \cdot 9{,}81\,m/s^2 = 1{,}312\,N$$

Angular velocity:

$$\omega_{wheel} = 2 \cdot \Pi \cdot n_{wheel} = 2 \cdot \Pi \cdot 1{,}875\exp(-02)/s = 0{,}118/s$$

Angular acceleration:

$$\alpha_{wheel} = \frac{2 \cdot \Pi \cdot n_{wheel}}{\Delta t} = \frac{2 \cdot \Pi \cdot 1{,}875\exp(-02)/s}{1s} = 0{,}118\,\frac{rad}{s^2}$$

Acceleration torque:

$$T_{a(wheel)} = J_{wheel} \cdot \alpha_{wheel} = 3{,}0\exp(-04)\,kg \cdot m^2 \cdot 0{,}118\,\frac{rad}{s^2} = 3{,}54\exp(-05)\,Nm = \mathbf{3{,}53\exp(-02)\,mNm}$$

Resulting force related to the wheel radius:

$$F_{res} = \frac{T_{a(wheel)}}{r_{wheel}} = \frac{3{,}53\exp(-02)\,mNm}{0{,}5 \cdot 80{,}277\,mm} = \mathbf{8{,}81\exp(-04)\,N}$$

4.2 Bearings

4.2.1 Bearings of the worm

The dimensions of the worm and the equations to divide the gravity force to the bearings L1 and L2 are equal to point 2.2.1.

Equations to calculate the radial, tangential and axial force components:

Radial force:

$$F_{L1(radial)} = F_{r1} \cdot \left(\frac{a.1}{(a.1 + a.2)} \right) + F'_{L1} = 3{,}22 \exp(-04) N \cdot \left(\frac{25mm}{(25mm + 25mm)} \right) + 0{,}160 N = 0{,}160 N$$

$$F_{L2(radial)} = F_{r1} \cdot \left(\frac{a.2}{(a.1 + a.2)} \right) + F'_{L2} = 3{,}22 \exp(-04) N \cdot \left(\frac{25mm}{(25mm + 25mm)} \right) + 0{,}156 N = 0{,}157 N$$

Tangential force:

Distance a.1 is equal to a.2, therefore $F_{L1(tangential)} = F_{L2(tangential)}$

$$F_{L1(tangential)} = F_{L2(tangential)} = F_{t1} \cdot \left(\frac{a.2}{(a.1 + a.2)} \right) = 1{,}04 \exp(-04) N \cdot \left(\frac{25mm}{(25mm + 25mm)} \right) = 5{,}2 \exp(-05) N$$

Axial force:

Distance a.1 is equal to a.2, therefore $F_{L1(axial)} = F_{L2(axial)}$

$$F_{L1(axial)} = F_{L2(axial)} = F_{a1} \cdot \left(\frac{dm1}{2 \cdot (a.1 + a.2)} \right) = 8{,}81 \exp(-04) N \cdot \left(\frac{12{,}043mm}{2 \cdot (25mm + 25mm)} \right) = 1{,}06 \exp(-04) N$$

Resulting forces of the bearings L1 and L2:

$$F_{L1} = \sqrt{(F_{L1(radial)} + F_{L1(axial)})^2 + F^2_{L1(tangential)}} = \sqrt{(0{,}160 N + 1{,}06 \exp(-04) N)^2 + 5{,}2 \exp(-05) N^2} = 0{,}16 N$$

$$F_{L2} = \sqrt{(F_{L2(radial)} - F_{L2(axial)})^2 + F^2_{L2(tangential)}} = \sqrt{(0{,}157 N - 1{,}06 \exp(-04) N)^2 + 5{,}2 \exp(-05) N^2} = 0{,}16 N$$

Approximated calculation of the friction torque without considering of the lubricant:

$$M_{friction(L1)} = F_{L1} \cdot \mu \cdot \frac{d}{2} = 0{,}16 N \cdot 0{,}0015 \cdot \frac{5mm}{2} = 6{,}0 \exp(-04) mNm$$

$$M_{friction(L2)} = F_{L2} \cdot \mu \cdot \frac{d}{2} = 0{,}16 N \cdot 0{,}0015 \cdot \frac{5mm}{2} = 5{,}9 \exp(-04) mNm$$

$$M_{friction(L1+L2)} = M_{friction(L1)} + M_{friction(L2)} = \mathbf{1{,}19 \exp(-03) mNm}$$

4.2.2 Bearings of the wheel

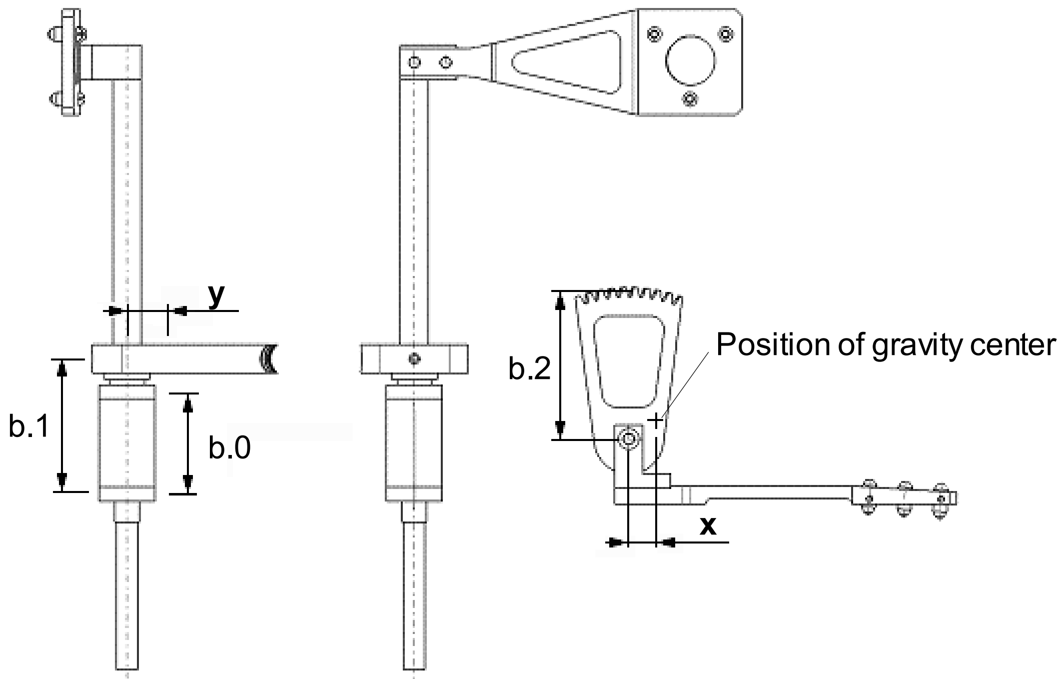

Fig. 9: Dimensions, effective forces and torques on the slit lock fork

Distance	b.0	mm	29
Distance	b.1	mm	9,5
Distance	b.2	mm	40,1385
Inside diameter of bearing	d	mm	8

Acting forces to the bearings L3 and L4:

Radial force:

$$\sum F_{radial}=0;\quad -F_{r2}+F_g-F_{L3(radial)}+F_{L4(radial)}=0$$

$$F_{L3(radial)}=\frac{F_g\cdot(X)+F_{t1}\cdot b.2-F_{r1}\cdot(b.0+b.1)}{b.0}$$

$$F_{L3(radial)}=\frac{1{,}312\,N\cdot 3{,}49\,mm+1{,}04\exp(-04)\,N\cdot 40{,}1385\,mm-3{,}22\exp(-04)\,N\cdot 38{,}5\,mm}{29mm}=0{,}158\,N$$

$$F_{L4(radial)}=F_{L3(radial)}+F_{r1}=0{,}158\,N+3{,}22\exp(-04)\,N=0{,}158\,N$$

Axial force:

$$F_{L3(axial)}=F_{L4(axial)}=F_{t1}+F_g=1{,}04\exp(-04)\,N+1{,}312\,N=1{,}312\,N$$

Resulting forces of the bearings L3 and L4:

$$F_{L3} = \sqrt{F^2_{L3(axial)} + F^2_{L3(radial)}} = \sqrt{1{,}312\,N^2 + (0{,}158)^2} = 1{,}321\,N$$

$$F_{L4} = \sqrt{F^2_{L4(axial)} + F^2_{L4(radial)}} = \sqrt{1{,}312\,N^2 + 0{,}158\,N^2} = 1{,}321\,N$$

Approximated calculation of the friction torque without considering of the lubricant:

$$M_{friction(L3)} = F_{L3} \cdot \mu \cdot \frac{d}{2} = 1{,}321\,N \cdot 0{,}0015 \cdot \frac{8mm}{2} = 7{,}93\,\exp(-03)\,mNm$$

$$M_{friction(L4)} = F_{L4} \cdot \mu \cdot \frac{d}{2} = 1{,}321\,N \cdot 0{,}0015 \cdot \frac{8mm}{2} = 7{,}93\,\exp(-0{,}3)\,mNm$$

$$M_{friction(L3+L4)} = M_{friction(L3)} + M_{friction(L4)} = 1{,}59\,\exp(-02)\,mNm$$

4.3 Torques, Forces and Efficiency of the worm gear

Resulting force of in relation of the wheel radius:

F_{res} is the basis force to calculate the further forces which are acting on the roll point of the worm gear. F_{res} is equal to F_{t2} and F_{a1}.

$$F_{res}=F_{t2}=F_{a1}=8{,}81\,\exp(-04)N$$

Tangential force F_{t1}:

$$F_{t1}=F_{a1}\cdot\tan(\gamma m+\rho')=8{,}81\exp(-04)N\cdot\tan(4{,}7631+2)=1{,}04\exp(-04)N$$

Backup force F'_e:

$$F'_{e1}=\sqrt{F_{a1}^2+F_{t1}^2}=\sqrt{8{,}81\exp(-04)N^2+1{,}04\exp(-04)N^2}=8{,}87\exp(-04)N$$

Projection of normal force F_{n1}:

$$F'_{n1}=F'_{e1}\cdot\cos(\gamma m)=8{,}87\exp(-04)N\cdot\cos(4{,}7631)=8{,}84\exp(-04)N$$

Normal force F_{n1}:

$$F_{n1}=\frac{F'_{n1}}{\cos(\alpha 0)}=\frac{8{,}84\exp(-04)N}{\cos(20)}=9{,}40\exp(-04)N$$

Radial force F_{r1}:

$$F_{r1}=F_{n1}\cdot\sin(\alpha 0)=9{,}40\exp(-04)N\cdot\sin(20)=3{,}22\exp(-04)N$$

Friction force F_R:

Friction coefficient of stainless steel and bronze: $\mu=0{,}180$

$$F_R=F_{n1}\cdot\mu=9{,}40\exp(-04)N\cdot 0{,}180=1{,}69\exp(-04)N$$

Component of the friction force which is perpendicular to the worm axis F'_R:

$$F'_R=F_R\cdot\cos(\gamma m)=1{,}69\exp(-04)N\cdot\cos(4{,}7631)=1{,}69\exp(-04)N$$

Static friction moment T_{sf} of the worm:

$$T_{sf}=F'_R\cdot\frac{1}{2}\cdot dm1=1{,}69\exp(-04)N\cdot\frac{1}{2}\cdot 12{,}043mm=\mathbf{1{,}02\exp(-03)mNm}$$

Moment of inerta of the stepper motor and the Flexbeam-2:

Phytron stepper motor VSS 32.200.1,2	J	kg·m²	1,0exp(-06)
Flexbeam-2 / Type: PSMR10	J	kg·m²	2,9exp(-08)

Accelerations torque of the stepper motor:

$$T_{a(rotor)} = J_{worm} \cdot \alpha_{worm} = 1{,}0\,\exp(-06)\,kg \cdot m^2 \cdot 9{,}42 \frac{rad}{s^2} = 9{,}42\,\exp(-06)\,Nm = \mathbf{9{,}42\,exp(-03)\,mNm}$$

Accelerations torque of the Flexbeam-2:

$$T_{a(flexbam-2)} = J_{worm} \cdot \alpha_{worm} = 2{,}9\,\exp(-08)\,kg \cdot m^2 \cdot 9{,}42 \frac{rad}{s^2} = 2{,}73\,\exp(-07)\,Nm = \mathbf{2{,}73\,exp(-04)\,mNm}$$

4.4 Efficiency of the worm gear

The Efficiency of the slit lock fork worm gear is equal to point 2.4 in this report. It was accepted that there is no significant difference between a gear ratio of i = 120 and i = 80.

4.5 Conclusion

$$T_{start} = T'_{a(wheel)} + T_{sf} + T_{a(worm)} + T_{a(rotor)} + M_{friction(L1+L2)} + T_{a(flexbeam-2)}$$

$$T'_{a(wheel)} = \frac{T_{a(wheel)} + M_{friction(L3+L4)}}{i \cdot \eta_{12}} = \frac{3{,}54\,\exp(-02)\,mNm + 1{,}59\,\exp(-02)\,mNm}{80 \cdot 0{,}65} = \mathbf{9{,}88\,exp(-04)\,mNm}$$

The value of the calculated starting torque, as shown in the following result, is required to accelerate the worm gear of the slit lock fork. This result describes the movement of the fork in the moment before contacting the slit mount:

$$T_{start} = \mathbf{1{,}78\,exp(-02)\,mNm}$$