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Effect of adaptive telescope mirror dynamics on the residual of atmospheric turbulence correction

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ABSTRACT

In the present report we quantify the residual error of the correction of the atmospheric turbulence depending on the dynamical response of an Adaptive Telescope Mirror. The mirror is modeled having modes in closed loop with mass-spring-damper response. The equivalent transfer function is used to filter the atmospheric disturbance power spectrum and obtain the residual error.

Modification Record

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Abbreviations, acronyms and symbols

Symbol	Description
ATM	Adaptive Telescope Mirror
PSD	Power Spectral Density

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1 ATM as a temporal filter

In the hypothesis reported in Ref. [1], the Plant Transfer Function (PTF) $M(\mathbf{k},s)$ of the thin shell is given by

(1) $$M(\mathbf{k},s)=\frac{1}{ms^2+\gamma s+DA|\mathbf{k}|^4},$$

where $\mathbf{k}=(k_x,k_y)$ is the spatial frequency vector and $s=i\omega$ is the Laplace temporal frequency, m is the mass per actuator, γ is the damping of the trapped air per actuator, D is the flexural rigidity of the shell and A is the area per actuator. Assuming that the same local control loop for each actuator is a position-velocity scheme as shown in Fig. 1 with a proportional gain G in the position error branch and a derivative gain K in the velocity feedback (see Ref. [2] for more details), the Closed-Loop Transfer Function (CLTF) is given by:

(2) $$CLTF(\mathbf{k},s)=\frac{GH(s)}{ms^2+[\gamma+KH(s)]s+[DA|\mathbf{k}|^4+GH(s)]},$$

where the contribution of all the other terms (ADC, DAC, current driver and actuator CD, capacitive sensor S, linearization calculation LIN and computational filter delay D) have been combined in the transfer function $H(s)$. The glass sheet dynamics is dominated by the low-order spatial frequencies introducing most of the delay with respect to the other component [1], so we will consider $H(s)\approx 1$. Considering the previous assumptions the CLTF becomes:

(3) $$CLTF(\mathbf{k},s)=\frac{G}{ms^2+(\gamma+K)s+(DA|\mathbf{k}|^4+G)},$$

representing the response of a mass-spring-damper system. The static error (s=0) is compensated using the force Feed-Forward technique for modes having $DA|\mathbf{k}|^4\gtrsim G$. Moreover the response of the system is limited by the low spatial frequencies as discussed in Ref. [1], so, for our purposes, we can consider the conservative case with $DA|\mathbf{k}|^4\ll G$.

The error of the ATM correction is given by the Error Transfer Function $\mathrm{ETF}(\mathbf{k},s)=1-\mathrm{CLTF}(\mathbf{k},s)$:

(4) $$ETF(\mathbf{k},s)=\frac{ms^2+(\gamma+K)s}{ms^2+(\gamma+K)s+G},$$

or

(5) $$|ETF(i\omega)|^2=\frac{[(2\eta\omega_0)^2-\omega^2]\omega^2}{(\omega_0^2-\omega^2)^2+(2\eta\omega_0\omega)^2}=\frac{[(2\eta)^2-(\omega/\omega_0)^2](\omega/\omega_0)^2}{[1-(\omega/\omega_0)^2]^2+(2\eta\omega/\omega_0)^2},$$

where $2\eta\omega_0=(\gamma+K)/m$ and $\omega_0^2=G/m$. The parameters η and ω_0 represent, respectively, the normalized damping and the resonant frequency of the equivalent mass-spring-damper system at closed-loop:

(6) $$CLTF(i\omega)=\frac{\omega_0^2}{\omega_0^2-\omega^2+2i\eta\omega_0\omega},$$

giving the well-known step response

(7) $$1-\frac{\omega_0}{\omega_d}e^{-\eta\omega_0 t}\cos(\omega_d t-\Theta)$$

where $\omega_d=\omega_0\sqrt{1-\eta^2}$ and $\Theta=\sin^{-1}\eta$.

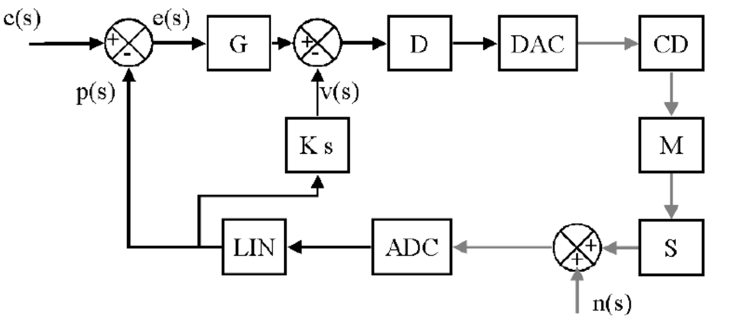

Fig. 1 Scheme of the control loop of a single actuator (Feed-Forward input is not shown).

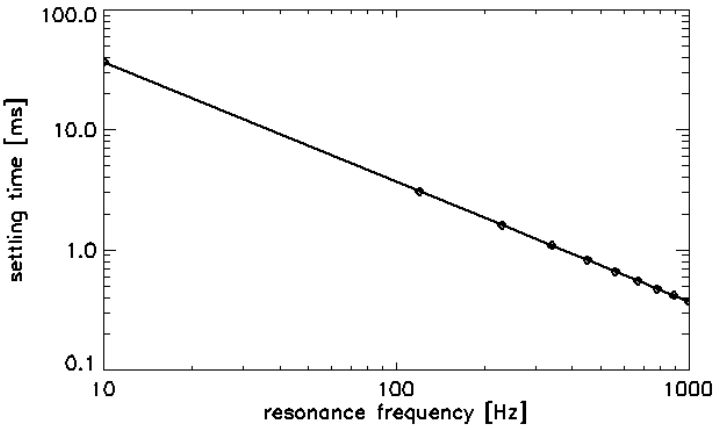

Fig. 2 Relationship between resonance frequency of a mass-spring-damper system and settling time to a step excitation in case of normalized damping equal to 0.59 (corresponding to 10% overshoot).

In literature the ATM response has been usually given and characterized in terms of settling time[1] and overshoot. The typical threshold of 10% overshoot is usually considered corresponding to $\eta \geq 0.59$. For such value of overshoot the relationship between the resonance frequency $\nu_0 = \omega_0/2\pi$ and the settling time can be evaluated numerically from Eq. (7) giving the plot in Fig. 2.

2 Residual of atmospheric turbulence correction

Ref. [3] reports the relationship between the residual error temporal PSD $w(\nu)$ and the spatial PSD $W_\phi(f_x, f_y)$ of the phase when a wind V along the x direction is present:

(8) $$w(\nu) = \frac{1}{V}\int_{-\infty}^{+\infty} |ETF(i2\pi\nu)|^2 W_\phi\left(\frac{\nu}{V}, f_y\right) df_y$$

and the rms σ_ϕ residual error is given by:

(9) $$\sigma_\phi^2 = 2\int_{-\infty}^{+\infty} w(\nu)\, d\nu$$

For instance, let us consider the Kolmogorov spectrum $W_\phi(f) = 0.023/\left(r_0^{5/3} f^{11/3}\right)$ and a pure delay filter for which $|ETF(i\omega)|^2 = \left|1 - e^{-i\omega\Delta t}\right|^2 = 2[1 - \cos(\omega\Delta t)]$. Combining Eq.(8) and (9) we have:

(10) $$\sigma_\phi^2 = 2\int_{-\infty}^{+\infty} \frac{1}{V}\int_{-\infty}^{+\infty} 2[1 - \cos(2\pi\nu\Delta t)] \frac{0.023}{r_0^{5/3}\left[\left(\frac{\nu}{V}\right)^2 + f_y^2\right]^{11/6}} df_y\, d\nu = \left[\frac{\Delta t}{0.31 r_0/V}\right]^{5/3}$$

obtaining the well-known formula defining the correlation time as $\tau_0 = 0.31 r_0/V$.

In the general case we have the Von Karman spectrum:

(11) $$W_\phi(f) = \frac{0.023}{r_0^{5/3}\left(f^2 + 1/L_0^2\right)^{11/6}}$$

that combined with the above formulas gives

(12) $$\sigma_\phi^2 = 2\int_{-\infty}^{+\infty} \frac{1}{V}\int_{-\infty}^{+\infty} \frac{\left[(2\eta)^2 - \left(\frac{\nu}{\nu_0}\right)^2\right]\left(\frac{\nu}{\nu_0}\right)^2}{\left[1 - \left(\frac{\nu}{\nu_0}\right)^2\right]^2 + \left(\frac{2\eta\nu}{\nu_0}\right)^2} \frac{0.023}{r_0^{\frac{5}{3}}\left[\left(\frac{\nu}{V}\right)^2 + f_y^2 + \frac{1}{L_0^2}\right]^{\frac{11}{6}}} df_y\, d\nu$$

$$= 2\frac{0.023}{V\, r_0^{5/3}} \int_{-\infty}^{+\infty} \frac{[(2\eta)^2 - (\nu/\nu_0)^2](\nu/\nu_0)^2}{[1 - (\nu/\nu_0)^2]^2 + (2\eta\nu/\nu_0)^2} \left[\left(\frac{\nu}{V}\right)^2 + \frac{1}{L_0^2}\right]^{-4/3} d\nu$$

[1] We define the settling time as the time interval between the start of the command and the time after which the position response is steadily within ±10% of the command.

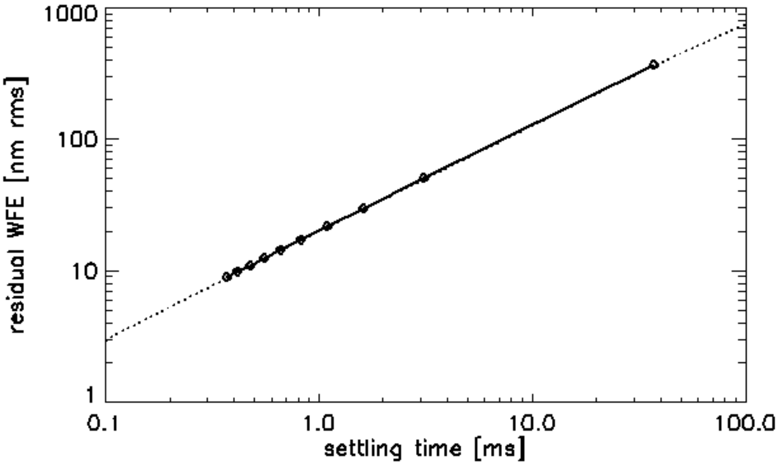

Fig. 3 upper limit of residual wavefront rms error due to mirror dynamics with respect to maximum modal settling time for a mirror with step-response-overshoot$\leq$ 10%. $r_0 = 12$ cm at 500nm (0.85 arcsec of seeing), L_0=50 m, $\tau_0 = 2.5$ ms (15 m/s of wind speed). The dashed line corresponds to a pure delay system having a delay equal to one half of the settling time.

The above integral has to be integrated numerically. The case of $r_0 = 12$ cm at 500nm (0.85 arcsec of seeing), L_0=50 m, $\tau_0 = 2.5$ ms (15 m/s of wind speed) and $\eta = 0.59$ (10% overshoot) is shown in Fig. 3 as function of the settling time (considering the conversion between resonant frequency ν_0 and settling time of Fig. 2). The same figure reports also the residual wavefront error for a pure delay system having a delay equal to one half of the plotted settling time. The exact superimposition of the two curves shows that an ATM system with 10% overshoot and settling time ST corresponds to a system with pure delay $\Delta t = ST/2$.

3 References

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