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Gas heat conduction in molecular regime

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Abstract

An infrared astronomical instrument is normally build inside a dewar. To evaluate the gas component of thermal exchange for such a system, we collect here the relevant results from gas kinetic theory from [1][2].

1 Basic computation

The magnitude of gas conduction depends on the interaction between the gas pressure level P and the characteristic void size `d` in terms of continuum or non-continuum behavior. The regimes of non-continuum behavior gas are characterized in terms of the Knudsen number K_N, defined as the ratio of the mean free path λ of molecules to a characteristic length `d` that is usually taken to be the width of the gas layer, i.e.:

$$K_N = \frac{\lambda}{d}$$

Where the mean free path (MFP) can be estimated from kinetic theory as

$$\lambda = \frac{1}{\sqrt{2} n \pi \sigma^2}$$

where n is the molecular number density and σ is the molecular diameter.
This formula, using the perfect gas law, can be written as

$$\lambda = \frac{k_B T}{\sqrt{2} P \pi \sigma^2}$$

where k_B is the Boltzmann constant and P is the gas pressure.

Using the previous formula, we calculate the MFP for nitrogen and helium at the pressure of $10^{-5} mbar (= 10^{-3} Pa)$ and at the temperature effective of

$$T_{eff} = \frac{283 + 79}{2} = 181K$$

$$K_B = 1.38 \cdot 10^{-23} J K^{-1}$$

$$\sigma_{N_2} = 310 pm$$

$$\sigma_{He} = 280 pm$$

Substituting the previous values we obtain:

$$\lambda_{N_2} = 5.6m$$

$$\lambda_{He} = 6.8m$$

The four different heat-transfer regimes are the continuum regime ($K_N < 0.001$), temperature jump regime ($0.001 < K_N < 0.1$), transition regime ($0.1 < K_N < 10$) and free molecule regime ($K_N > 10$). All four regimes are encountered for a given system as the gas pressure is reduced. The effective conductivity is a maximum for continuum conditions and decreases across the whole range of increasing K_N, at a given value of `d`.

Another physical effect which arises is the extent of thermal accommodation of the molecules to the surface temperature level, as they interact during each encounter. This effect is quantitatively described in terms of a coefficient of thermal accommodation α, which is the measure of the efficiency of energy exchange between a gas stream and a solid surface:

$$\alpha = \frac{E_i - E_r}{E_i - E_s}$$

where E_i is the incident energy flux, E_r is the reflected energy flux and E_s is the reflected energy flux obtained if the molecules are in thermal equilibrium with the surface.

Experimental studies have measured the thermal accommodation coefficient in a variety of geometries and over a wide range of gas-surface combinations. The data show that accommodation depends on the composition and the temperature of the gas and the surface, on the gas pressure, and on the state of the surface (roughness, gas adsorption). Experimental values are normally around unity, but measures range from 0.01 to nearly unity. The smaller values are observed for lower-molecular-weight gases striking clean surfaces. Near unity values are observed for higher-molecular-weight-gases striking contaminated or roughened surfaces.

If we calculate the Knudsen number for our case we obtain with $d = 10cm$:

$$K_{N N_2} = 5.6/0.1 = 56$$

and

$$K_{He} = 6.8/0.1 = 68$$

So we can affirm we are working in free-molecule heat-conduction regime and in this case the thermal energy flux is:

$$Q_{FM} = [\frac{1}{\alpha_1} + (\frac{r_1}{r_2})^b(\frac{1}{\alpha_2} - 1)]^{-1}[\frac{\gamma+1}{\gamma-1}]P(T_2 - T_1)\sqrt{\frac{R}{8MT\pi}}$$

Here b is 0 for parallel plates, 1 for coaxial cylinder and 2 for concentric spheres. T_2 and T_1 denote the temperatures of the surfaces of radii r_2 and r_1 respectively, where $r_2 > r_1$, P is the gas pressure, R is the molar gas constant and γ is the specific heat ratio of the gas, M is the molecular weight of the gas and $T = \frac{T_1+T_2}{2}$.
This heat flux is independent of the spacing `d` between the surfaces in free-molecule transport. Here we calculate the thermal energy flux for nitrogen and helium species, using the next values:

$$\gamma_{N_2} = 1.47$$

$$\gamma_{He} = 1.66$$

$$R = 8.31JK^{-1}mol^{-1}$$

$$M_{N_2} = 28gmol^{-1}$$

$$M_{He} = 4gmol^{-1}$$

$$r_2 = 1620mm$$

and using for both accommodation thermal coefficient α_1 and α_2 values between 0.8 and 1.

$$\alpha = [\frac{1}{\alpha_1} + (\frac{r_1}{r_2})^b(\frac{1}{\alpha_2} - 1)]^{-1} = [\frac{1}{0.8} + \frac{1610}{1620}(\frac{1}{0.9} - 1)]^{-1} \simeq 0.8$$

To get a conservative estimate we put $\alpha = 1$.

$$Q_{FMN_2} = \frac{2.47}{0.47} \cdot 10^{-3} \cdot 205 \cdot \sqrt{\frac{8.31}{8 \cdot 3.14 \cdot 28 \cdot 10^{-3} \cdot 181}} \simeq 0.3W/m^2$$

$$Q_{FMHe} = \frac{2.66}{0.66} \cdot 10^{-3} \cdot 205 \cdot \sqrt{\frac{8.31}{8 \cdot 3.14 \cdot 4 \cdot 10^{-3} \cdot 181}} \simeq 0.6W/m^2$$

References

[1] "Heat Conduction and Mass Diffusion", B. Gebhart, New York, 1993

[2] "The Infrared Handbook", W.L.Wolfe and G.J.Zizzis, Washington, 1978-1989