SKA Project Series
Effects of the bandpass slope on the SKA Low quantization

G. Comoretto[1]

[1]INAF - Osservatorio Astrofisico di Arcetri

Arcetri Technical Report N° 7/2013

Abstract

The SKA-LOW array observes a region of the radio spectrum where the astronomical signal spectral density varies by about two orders of magnitude. We study the effects of ADC quantization on a noise-like signal with a very steep spectrum, both analytically and by simulations. The only effect observed is the introduction on a additive quantization noise, with a spectrum well described by a flat Gaussian noise uncorrelated with the digitized signal. The total power in this noise corresponds to 1/12 of the squared quantization step, that is usually negligible even in the spectral regions where the input power density is lower.

1 Introduction

The SKA low telescope observes a spectral region ranging almost 3 octaves, The spectrum of the sky emission has a very steep slope, with S_ν proportional to $\nu^{-2.55}$. As a consequence, the spectral density of the signal seen by the digitizer spans almost a factor 100, or 20 dB, at the two extremes of the observed band.

This has caused concerns about possible inaccuracies and loss of dynamic range when such a "colored" signal is quantized by a limited resolution quantizer (e.g. 8 bit). A whitening filter has been proposed [3] to improve the spectral flatness of the signal to be digitized. Here we will analyze in detail possible problems due to the non white spectrum of the input signal, and address the necessity of a whitening network prior to quantization.

This will be performed analytically, in chapter 3, and by simulation, in chapter 4.

2 Signal description

The astronomical signal due to diffuse and background emission, in the spectral region of interest, can be described by a power law. The equivalent noise temperature, in K, is given by (see for example [1]):

$$S(\nu) = 125\,10^6 \left(\frac{1GHz}{\nu}\right)^{2.55} \tag{1}$$

At the two extremes of the SKA-low band, $\nu_l = 50$ MHz and $\nu_h = 350$ MHz, we have $S(\nu_l) = 5800$ K, $S(\nu_h) = 41$ K.

The receiver adds a roughly white noise to the astronomic signal, with a constant $S(\nu) = T_r$ that is not yet specified. We assumed, optimistically, $T_r = 40$ K [2]. A higher T_r improves the flatness of the band, so this represents a worse case scenario.

In the simulation we assumed that the astronomic signal is negligible below ν_l, and we added a strong spectral feature at mid band. The average signal equivalent temperature, including receiver noise, results 516 K.

The quantization has been assumed ideal (regular, equispaced thresholds), with 8 bit quantization. Non ideal quantization increases noise level, so the equivalent number of bits (ENOB) should be used instead, and the noise increased accordingly-

3 Mathematical analysis

Quantization basically introduces a noise with a RMS amplitude equal to $\sqrt{1/12}$ of the quantization step, and with typically a white spectrum. This noise adds to the radiometric and receiver noise, that in our case has a strong dependence on the input frequency. The quantization noise spectrum can be assumed flat (white noise) if it is uncorrelated between two successive samples, i.e. if the typical variation of the input voltage among samples is larger than a ADC step (ADU). This may not be false if the input signal is dominated by low frequency components (as it is in our case) and the signal amplitude is comparable with the quantization step. Simulations show, however, that the quantization noise spectrum is very flat, even for signal RMS amplitude of only 5 ADU.

To compare the two noises, it is necessary to determine the scale of the input signal. The signal RMS amplitude is proportional to the square root of the average $S(\nu)$, i.e.

$$S_a = \frac{1}{B}\int_B S(\nu)d\nu \tag{2}$$

If $S(\nu) = a\nu^{-k}$, extending from ν_1 to ν_2, then

$$S_a = \frac{a}{\nu_2 - \nu_1}\int_{\nu_1}^{\nu_2} \nu^{-k} d\nu = \frac{S(\nu_1)}{k-1}\left(\frac{1-\left(\frac{\nu_1}{\nu_2}\right)^{k-1}}{\frac{\nu_2}{\nu_1}-1}\right) \tag{3}$$

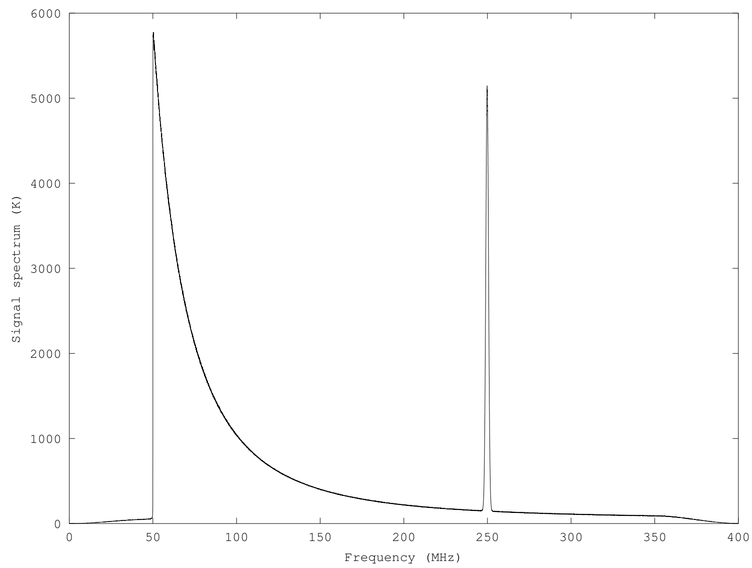


Figure 1: Spectrum of the test signal used in the simulations

For the frequency range of SKA-low $S(\nu_)$ = 5800 K, $k = 2.55$, $\nu_2/\nu_1 = 7$ and $S_a = 0.102\,S(\nu_1) =$ 592 K. Adding a white noise with $T = 50$ K, and considering a total band of 400 MHz, the average equivalent signal temperature is about 500 K, corresponding to a RMS amplitude of about 22 $K^{1/2}$.

The RMS signal amplitude must not exceed 1/8 the ADC range, i.e. 256/8 = 32 ADU for an 8 bit ADC, to prevent nonlinearities due to signal clipping[4]. The actual amplitude should be lower than this limit to avoid corruption of the signal due to RFI. If the RMS amplitude corresponds to 16 ADU, the ADC scale is 1.9 K/ADU^2. For an ideal ADC, the quantization noise of 1/12 ADU^2 thus corresponds to 0.12K of excess instrumental (receiver) noise. This must be compared to the system noise temperature in the coldest portion of the spectrum, that is about 90 K. The sensitivity loss is 0.12/90 =0.13%, i.e. equivalent to a loss of one antenna element every 750.

The excess noise increases quickly for a non ideal ADC. If the ADC has less than 8 bits of ENOB the excess noise doubles every 0.5 bit. Absolute numbers are however always small.

The effect of introducing an equalization network would effectively shape the quantization noise to mimic the input bandwidth. For the optimal case, when the spectrum at the ADC input is completely flat, the quantization noise would be the same than for a signal with the average system temperature. This is just a factor of 5 in our case, i.e. the ratio between the minimum system temperature, 90 K, and the average one, 500 K. The same factor can be achieved by using an ADC with one more ENOB.

4 Numeric simulations

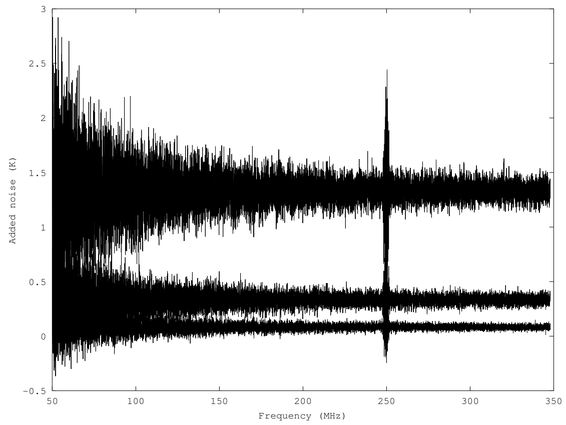
To test this hypothesis we ran a set of simulations, using a test vector consisting of a Gaussian noise with predefined spectral content, and various sampling options.

The spectrum consists of a uniform white noise, with spectral density of 40 K, plus a colored spectrum with $T(\nu) = 5800K(\nu/\nu_1)^{-2.55}$, for $\nu_1 = 50$ MHz. A spectral line (always composed only by Gaussian noise) with intensity of 5000 K at 205 MHzi has been added to investigate the effect of localized spectral features. Due to program limitation there is an abrupt jump in the spectrum below ν_1, that affects the data. The corresponding features at this frequencies are thus artefacts and only the portion of the spectrum in the SKA-low passband has been considered. The total average signal temperature is 516 K, corresponding to a signal RMS amplitude of 22.72 $K^{1/2}$.

The generated signal is composed of $2,10^9$ samples, that has been quantized to 8 bit samples. For simplicity we assumed three cases, respectively with a scale of 1, 0.5 and 0.25 $K^{1/2}$/ADU, or a RMS value of 22.72, 11.36 and 5.68 ADU respectively. These values are close to the practical limits of the optimal amplitude range at the ADC input.

The resulting real (unquantized) and quantized signals were analyzed using a FFT with a total of 32768 frequency points. Noise has been computed comparing the quantized spectra with the original template spectrum, and with the spectrum of the unquantized signal. The spectrum of the unquantized signal is shown in fig. 1, and is basically identical to the spectra of the quantized signals.

The difference between the quantized and unquantized spectra are shown in fig. 2, from top to bottom relative to the signals with RMS amplitude of 5.68, 11.36 and 22.72 ADU. Left plot shows spectra computed using 2^{15} channels ($\Delta\nu = 12.2$ KHz), and right plot spectra averaged over 100 channels, to better show the constant added noise. The corresponding excess noise is equal to the theoretic values, computed in the previous chapter, of 1.33 K, 0.33 K and 83 mK.



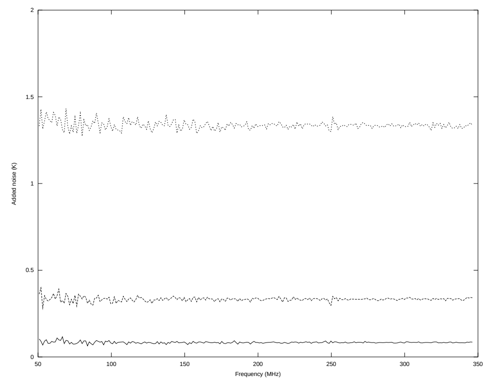


Figure 2: Difference between the spectrum of the unquantized signal and the three quantized signals. Left, $\Delta\nu = 12.2$ KHz, right $\Delta\nu = 1.22$ MHz. The RMS amplitude of the quantized signals are, from bottom to top, 5.68, 11.36 and 22.72 ADU

An independent way to assess the extra noise is computing the variance of the difference between the quantized and unquantized spectra. If the unquantized spectrum is $S_u =< x_u^2 >$ and the quantized one is $S_q =< x_q^2 >= \left\langle (x_u + n_q)^2 \right\rangle$, with n_q the quantization noise, we have:

$$S_q - S_u = 2 \langle x_u n_q \rangle + < n_q^2 > \tag{4}$$

The difference averages to $< n_q^2 >$, that, as we have seen in fig. 2, is very close to the theoretical value. The cross product averages to zero if the two signals are uncorrelated, zero mean signals. This is not strictly true, as n_q is a function of x_u, but is a reasonable approximation if the signal spans several ADC levels. The variance of this quantity is, again assuming uncorrelated quantization noise:

$$V(S_q - S_u) = \frac{4 < x_u^2 >< n_q^2 >}{\tau \, \Delta B} \tag{5}$$

where τ is the integration length and ΔB is the effective noise bandwidth f the spectrometer.

In figure 3 we plotted the quantity $1/2(\tau \, \Delta B)\sqrt{V(S_q - S_u)}$, smoothed in frequency. This quantity is very close to the expected value of $\sqrt{< n_q^2 >}$, that for the three signal levels of our test vectors is 0.289, 0.577, and 1.155 $K^{1/2}$ respectively. The added noise is very flat, as the added radiometric noise, and does not show spectral features correlated with the input signal, even where the added noise is a significant part of the total radiometric noise, or in presence of strong spectral features.

5 Conclusions

The quantization of a signal composed of a Gaussian noise with an arbitrary spectrum can be described with very good accuracy as an additional source of noise. The quantization noise is white and uncorrelated

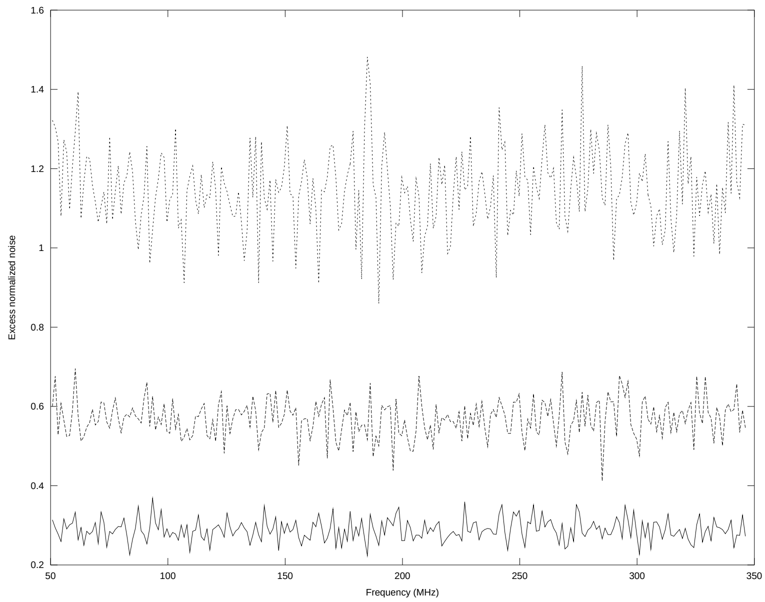


Figure 3: Normalized added noise between the spectrum of the unquantized signal and the three quantized signals. Scale is in $K^{1/2}$

to the signal to be quantized, even when the latter has a RMS amplitude of only 5 ADUs and spectral density spanning 20 dB. The only effect is thus a small increase in the total radiometric noise.

The strong spectral slope in the input signal causes the quantization noise to become more relevant in the spectral regions where the input power density is lower. As the quantization noise is proportional to the average input level, the increase in the relative quantization noise passing from a flat to a shaped spectrum is equal to the ratio of the minimum vs. average spectral density. This factor is about 5 in the example considered here. As the quantization noise for a 8 bit ADC is typically a factor of a few 1000 below the radiometric noise, this increase then produces an excess noise that is everywhere below 1%, providing that the ADC input level remains at least at 5-7 times the ENOB quantization step, and no equalization network is necessary.

References

[1] Bakker, Laurens, Bij de Vaate, Jan Geral: "SKA-low RF systems overview" (2011)

[2] G. Bianchi, F. Perini, J. Monari, C. Bortolotti, M. Roma: "Estimation of the requested ADC Dynamic Range for the SKA-AAlo core" *INAF–IRA Report* (2011)

[3] G. Bianchi, F. Perini: "Additional notes to: Estimation of the requested ADC Dynamic Range for the SKA-AAlo core" *INAF–IRA Report* (2013)

[4] G. Comoretto: "Sub-channel stitching and truncation errors in the ALMA Tunable Filterbank", *Arcetri Technical Report* (2004)