Theoretical Studies of Interstellar Dust

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"Sure it's beautiful, but I can't help thinking about all that interstellar dust out there."

Plan

- What can we calculate to compare to observations?
 - Interstellar Extinction

• Grain models: ingredients

- Interstellar Polarization
- Scattering Properties of Dust
- Heating/Cooling of Grains and Infrared Emission
- Rotational Dynamics and "Spinning Dust" Emission
- Magnetic Dipole Emission from Magnetic Dust?
- Grain Destruction in the ISM
- Grain Evolution in ISM of Milky Way and Other Galaxies

Some things I will not talk about:

- charging of interstellar grains
- role of grains in ionization balance
- heating by photoelectrons from grains
- deuteration of PAHs
- physics of grain alignment
- evolution of dust in dense clouds
- dynamics (and acceleration) of charged grains in interstellar shocks

I'm happy to discuss these or other dust physics matters any time – just stop by my office.

Dust Model Ingredients



Early chemists describe the first dirt molecule.

Dust in Diffuse Clouds (no ices present)

Essential Ingredients

- amorphous silicate material to account for 10μ m and 18μ m absorption
- Polycyclic aromatic hydrocarbon (PAH) material to account for IR emission features
- Additional carbonaceous material with significant aliphatic fraction to account for 3.4μ m feature
- Other stuff stardust SiC, diamond, etc. – appears to have sufficiently low abundance that it can be ignored at the present stage of modeling.

Extinction Curves

- Choose an observed extinction curve to reproduce
- specify components, e.g.,
 - population of silicate grains
 - population of carbonaceous grains
- Choose grain shape (e.g., oblate spheroid, axial ratio b/a)
- Fit extinction vs. λ by varying size distributions dn_j/da (e.g., j = silicate , carbonaceous)
- Fit polarization vs. λ by varying $f_j(a)$ = degree of alignment for grains of composition j, size a

Also:

- Adjust size distribution of PAHs so that single-photon heating reproduces observed IR emission.
- Satisfy abundance constraints.

Abundance Constraints



Solid symbols = C, Mg, Si, Fe (major grain constituents) (Draine 2011).

- Sightline to nearby star ζ Oph (O9.5V, $D = 112 \pm 3pc$) has superb high-resolution absorption-line spectroscopy.
- If ISM is assumed to have solar abundances, then "depletions" of elements like C, Mg, Si, Fe indicate incorporation into solids.
- Total "missing mass"/H mass = 0.0091 ± 0.0006
 - Challenge: to reproduce observed extinction without exceeding abundance constraints.

Size Distribution of Interstellar Grains

- Observe extinction curve from $\sim 2\,\mu\mathrm{m} 0.1\,\mu\mathrm{m}$
- Mathis et al. (1977) tried to reproduce average interstellar extinction curve using mixture of graphite and silicate spheres.
- Using non-parametric size distribution with upper and lower cutoffs, a_{\min} and a_{\max} , they found best-fit size distributions dn/da.
- Their best-fit size distributions were very close to power-laws!!
- Therefore MRN proposed using power-laws

 $\frac{1}{n_{\rm H}} \frac{dn_{\rm gra,sil}}{da} = A_{\rm gra,sil} a^p \quad \text{for } a_{\rm min} < a < a_{\rm max}$ p = -3.5 $a_{\rm min} \approx 0.005 \,\mu\text{m}$ $a_{\rm max} \approx 0.25 \,\mu\text{m}$

This is the famous "MRN" size distribution.

- $dn/da \propto a^{-3.5}$ has most of mass at large size end, most of area at small size end.
- $dn/da \propto a^{-3.5}$ is similar to size distribution of
 - $\Rightarrow p \approx -3.25$ for asteroids with 5 < D < 300 km (Bottke et al. 2005)
 - steady-state coagulation/fragmentation models (Dohnanyi 1969; Weidenschilling 1997; Tanaka et al. 1996, 2005)
- **Problem:** Because of PAHs, the MRN distribution can no longer be considered applicable to interstellar dust.
 - ♦ Substantial mass in ultrasmall dust grains: ~ 5% of total dust mass is in particles with < 10³ C atoms. This is *much* more than MRN extended to very small sizes.
 - PAHs contribute substantially to the UV extinction.
 - ◊ Non-PAH extinction not well-fitted by MRN distribution.

Modeling Extinction





- Calculation of $C_{\mathrm{ext}}(\lambda)$ is
 - easy if spheres are assumed
 - not so easy for spheroids
 - challenging for more complex shapes
- Models are not unique.



Extinction contributed by silicate and carbonaceous material in WD01 model.

- "Observed" extinction in IR ($\lambda > 1 \,\mu$ m) is not well-determined WD01 and ZDA04 models differ in what they adopt as "observed" IR extinction.
- In recent years there have been revisions to the "observed" extinction in the 3–8 μm region. Current models don't agree with observations in this region.



Mass distributions for different grain models: (a) WD01=Weingartner & Draine (2001) (b) ZDA04=Zubko et al. (2004); (c) DF09=Draine & Fraisse (2009).

figure from Draine (2011)

Dust Mass Distribution

- Models not unique, but general agreement on overall distribution of grain mass.
- Most of grain mass at $0.05 < a < 0.5 \,\mu\text{m}.$
- "Typical" (half-mass) grain radius $\sim 0.1 \, \mu {
 m m}$
- Size distribution is **not** a power-law.
- Significant mass in *a* < 1nm particles required to explain PAH emission.
- Models not consistent with claimed flux of $a > 0.3 \,\mu\text{m}$ particles entering heliosphere.

Regional Variations in Size Distribution

- Extinction curves are known to vary from one sightline to another.
- Denser regions tend to have
 - \diamond "flatter" extinction curves, i.e., higher values of $R_V \equiv A_V / E(B V)$
 - \diamond increased R_V is attributed to tilt in size distribution to decrease numbers of small particles, increase numbers of larger particles.
 - \$\\$ grain growth is presumably due partially to accretion of atoms from gas, but this is only a minor effect (because unless ices can form, most depletable species are already depleted in diffuse clouds)

 \diamond grain growth must be due primarily to coagulation.

• timescale for dust grain to collide with another dust grain is relatively short:

$$\tau_{dd} = \frac{1}{n_{\rm H} \Sigma_d \ (\Delta v)_{dd}} = 1 \times 10^7 \,\mathrm{yr} \left(\frac{30 \,\mathrm{cm}^{-3}}{n_{\rm H}}\right) \left(\frac{10^{-21} \,\mathrm{cm}^2/\mathrm{H}}{\Sigma_d}\right) \left(\frac{1 \,\mathrm{km \, s}^{-1}}{\Delta v_{dd}}\right)$$

- dust-dust velocity differences $\Delta v_{dd} \sim 1 \,\mathrm{km \ s^{-1}}$ are expected
 - radiation pressure and "recoil" effects can cause grains to drift through gas with speeds that depend on
 size and composition
 - ordinary fluid turbulence will give grains random velocities
 - \diamond MHD turbulence can pump energy into "orbital" motions of $\gtrsim 0.1\,\mu{\rm m}$ grains in diffuse clouds (Yan et al 2004)
- It is likely that coagulation modifies the grain size distribution. Presumably balanced by shattering in higher-velocity grain-grain collisions (Yan et al. 2004; Hirashita & Yan 2009; Hirashita et al. 2010)

Some Uncertainties

• Separate populations of silicate and carbonaceous grains?

- It is natural for "stardust" to be segregated (silicate grains vs. carbon grains) but interstellar grains may be heavily affected by coagulation processes.
- The 10 μm silicate feature is polarized therefore silicate-containing grains are aligned
- The $3.4 \,\mu\text{m}$ aliphatic C-H stretch shows no evidence of polarization consistent with separate, non-aligned, population of carbonaceous grains.
- But 3.4 μm feature is weak few studies of polarization
- Degree to which grain materials are segregated remains uncertain.

• Grain geometry?

- Compact grains?
- Fluffy grains resulting from agglomeration?
- Models with compact grains can reproduce observations.
 What about fluffy grains? We don't know needs to be studied.

Grain Geometry: Uknown

• Are interstellar grains fairly smooth and compact?



Presolar onion-like graphite grain (diameter $\sim 5 \,\mu{\rm m}$). Photo from S. Amari.

• Or are they typically loose aggregates of smaller particles, with a large "porosity"?



Two interplanetary dust particles collected from stratosphere (diameter $\sim 10 \,\mu m$). Elemental compositions similar to primitive meteorites: silicates + carbonaceous material.

Images courtesy E.K. Jessberger and Don Brownlee.

How Can We Determine Grain Geometry?

• Direct capture?

- Stardust mission: (probably) destructive capture
- Future mission: need to figure out how to capture incoming particles without destroying them...

• Try to reproduce extinction and polarization observations

- Compact grains: OK
- Fluffy grains we don't yet know
- Work in progress...

• Try to reproduce X-ray halos

- Compact grains: OK
- Fluffy grain models → more concentrated X-ray halos (Heng & Draine 2009). May be inconsistent with observations.
- Work in progress...

Heating and Cooling of Grains: Infrared Emission

For given starlight radiation field u_{ν} , and grain of given composition c and radius a, calculate the probability distribution function dP/dE for internal energy E:

- \diamond define N energy bins E_j (we use N = 500)
- \diamond calculate transition matrix T_{ji} = probability per unit time of transition $i \rightarrow j$. upward transitions due to photon absorption downward transitions due to photon emission
- \diamond Let P_j = probability that grain will be in bin j

 \diamond Then

$$\frac{d}{dt}P_j = \sum_{i \neq j} T_{ji}P_i - \sum_{k \neq j} T_{kj}P_j$$

♦ Find steady state solution

$$0 = \sum_{i \neq j} T_{ji} P_i - \sum_{k \neq j} T_{kj} P_j$$

with $\sum_{j} P_j = 1$.

 \diamond Repeat for many different sizes a.

Upward transition rates are calculated using absorption cross section $C_{abs}(\nu)$ and radiation field u_{ν} .

How to calculate downward transition rates

 $T_{ji} \quad j < i$?

See discussion in Draine & Li (2001).

A good approximation is to associate a temperature T_j with each energy bin E_j :

$$T_j = T$$
 for which $\langle E \rangle = E_j$.

Thermal approximation: assume that grain with energy E_j has emission spectrum

$$j_{\nu} = C_{\rm abs}(\nu) B_{\nu}(T_j)$$

where $B_{\nu}(T) =$ blackbody function.

Because energy bins have finite width, need to give some care to calculation of T_{ij} .

$$\implies (dP/dT)_{c,a}$$

A Day in the Life of 5 Interstellar Grains



• grain with 50 C atoms in local starlight (U = 1): ~1 absorption/100 days



Temperatures of "Classical" Grains

For large grains,

$$(dP/dT)_{c,a} \implies \delta(T - T_{\rm ss}(c,a))$$

Steady-state temperature $T_{ss}(c, a)$ is solution to heating = cooling

$$\int d\nu C_{\rm abs}(\nu) \, u_{\nu}c = \int d\nu C_{\rm abs}(\nu) \, B_{\nu}(T_{\rm ss})$$

If $C_{\rm abs} \propto
u^{eta} \propto \lambda^{-eta}$ in IR, then

$$\int d\nu \, C_{\rm abs}(\nu) \, B_{\nu}(T_{\rm ss}) \propto T_{\rm ss}^{(4+\beta)}$$

and

$$T_{
m ss} \propto u_{\star}^{1/(4+eta)}$$

If

$$C_{\rm abs} \propto a^2$$
 for starlight absorption $(a \gtrsim 0.1 \,\mu{\rm m})$
 $C_{\rm abs} \propto a^3$ for IR emission $(a \lesssim 10 \,\mu{\rm m})$

then

$$T_{
m ss} \propto u_{\star}^{1/(4+eta)} \; a^{-1/(4+eta)}$$

Bigger grains are slightly cooler.

Typically
$$\beta \approx 2 \Longrightarrow T_{\rm ss} \propto u_{\star}^{1/6} a^{-1/6}$$

Emission Spectra

Sum over compositions c, integrate over size a to get emission spectrum:

$$\begin{split} j_{\nu} &= \sum_{c} \int da \left(\frac{dn}{da} \right)_{c} \times \\ &\int dT \left(\frac{dP}{dT} \right)_{c,a} C_{\rm abs}(a,\lambda) B_{\nu}(T) \end{split}$$

 $\lambda \lesssim 15 \,\mu{
m m}$ emission spectrum (PAH features) independent of U for $U \lesssim 10^4$

emission following single-photon heating events



IR Emission: Models vs. Observations

(Aniano et al. 2012)



Dust-to-Gas Ratio in NGC 6946 at MIPS 160 resolution



How about Nearby Galaxies?100 μm IRAS/COBE Map of Sky (after zodi subtraction)Image credit: D. Finkbeiner



The Small Magellanic Cloud (SMC)



- Interstellar gas less enriched with heavy elements (C,N,O,...,Fe) formed in massive stars SMC metallicity ~25% "solar"
- Composition of dust appears to differ from dust in the Milky Way. 2175Å "bump" is weaker

Dust in the SMC: Excess 50–300 GHz Emission



- Photometry: Israel et al. (2010) and Planck Collaboration et al. (2011)
- $M_{\rm H}(SMC) \approx 4.8 \times 10^8 M_{\odot}$
- $Z(SMC) \approx 0.25 Z_{\odot}$
- $M_{\rm dust,max}(SMC) \approx 1.2 \times 10^6 M_{\odot}$
- After subtracting
 - synchrotron emission
 - free-free (bremstrahhlung)
 - chance upward fluctuation of CMB

Can dust model + starlight reproduce the observed emission?



- Model with acceptable mass of dust, but severe 50–200 GHz shortfall.
- Dust in SMC is more emissive at mm wavelenths than MW dust... why?
- At long wavelengths (particle size « λ), it is usually assumed that emission comes from *thermal fluctuations in the electric dipole moment*.

Perhaps this isn't the only source of emission from dust....

Magnetic Dipole Emission from Magnetic Dust

(Draine & Lazarian 1999; Draine & Hensley 2012b)

- Suppose much of the Fe is in magnetic material (e.g., metallic Fe, magnetite Fe₃O₄, or maghemite γ–Fe₂O₃)
- Lowest energy state of metallic Fe:
 - spins are parallel (magnetized),
 - magnetization $\vec{\mathbf{M}}$ is aligned with one of the crystal axes
- Excited state: spins parallel, but oriented away from crystal axis

- Oscillations in magnetization \rightarrow magnetic dipole emission
- Finite temperature → thermal magnetic dipole emission



Magnetic Dipole Absorption Cross Section for Fe Nanoparticles

(Draine & Hensley 2012b)



- Magnetization dynamics: use **Gilbert equation** (*not* Landau-Lifshitz eq. or Bloch-Bloembergen eq.)
- For metallic Fe: ferromagnetic resonance frequency depends on particle shape.
- Absorption depends on uncertain "Gilbert damping parameter" $\alpha_{\rm G}$. $\alpha_{\rm G} \approx 0.2$ may be realistic.

SMC Dust Models With Iron or Magnetite (Fe₃**O**₄) Nanoparticles



• $M_{\text{dust}} = 8.4 \times 10^5 M_{\odot}$ or $6.4 \times 10^5 M_{\odot}$ (both $< M_{\text{dust,max}} = 1.2 \times 10^5 M_{\odot}$)

- \bullet magnetic dipole emission dominates for $\nu \stackrel{<}{_\sim} 200\,{\rm GHz}$
- spinning dust component:
 - normal spectrum (peaking near 40 GHz)
 - has \sim expected strength (scaled with PAH abundance in SMC)

Rotational Dynamics: Spinning Dust



Rotational Dynamics: Spinning Dust Recent Refinements

- Will spinning dust emission be polarized? **NO:** pol < 1% at $\nu > 30$ GHz (Lazarian & Draine 2000)
- Factor of two correction to rotational excitation by photon emission (Ali-Haïmoud et al. 2009)
- Analytic solution to Fokker-Planck equation (Ali-Haïmoud et al. 2009)
- Polarized IR emission from PAHs in reflection nebulae (Sironi & Draine 2009)
- Allow for rotation around non-principal axis (Hoang et al. 2010; Silsbee et al. 2011)
- Include effects of high- ΔJ impacts with ions (Hoang et al. 2010)
- Wobbling for general asymmetric grains (moment-of-inertia tensor with 3 nondegenerate eigenvalues) (Hoang et al. 2011)
- Include internal relaxation (coupling of vibrational and rotational modes) with transient heating (Hoang et al. 2011)



Grain Destruction in the ISM

Principal Mechanisms for solid \rightarrow gas in SN Blastwayes

- Sputtering (removal of single atoms following impact of H or He ions)
- Vaporization in grain-grain collisions
- SN shock will destroy dust if $v_s \gtrsim 250 \text{ km s}^{-1}$. $Mv_s^2 \approx 2.7E$ (Sedov)

$$M = \frac{2.7 \times 10^{51} \,\mathrm{ergs}}{(250 \,\mathrm{km \, s^{-1}})^2} = 2200 M_{\odot}$$

• 1 SN/50 years:

$$\tau_{\rm dest} = \frac{M_{\rm ISM}}{2200 M_{\odot}/50 \,\mathrm{yr}} = 1.6 \times 10^8 \,\mathrm{yr}$$

Complications:

- additional (partial) destruction in lower-velocity shocks
- ISM is inhomogeneous (but does mix rapidly)
- SN are correlated
- $\tau_{\rm dest} \approx 4 \times 10^8 \, {\rm yr}$
- Problem revisited a number of times (e.g., Barlow 1978; Draine & Salpeter 1979; Dwek & Scalo 1979; Jones et al. 1994) with similar conclusions (but see Jones & Nuth 2011, for dissenting view).

Grain Destruction and Reformation in the ISM Mass Budget for the MW ISM



- ISM "lifetime" $M_{\text{ISM}}/(\text{SFR} - \dot{M}_{\text{in}} - \dot{M}_{\text{return}}) = 1.2 \times 10^{10} \,\text{yr}$
- $\tau_{\rm SFR} = M_{\rm ISM}/{\rm SFR} = 3.5 \times 10^9 \, {\rm yr}$
- $\tau_{\text{dest}} \equiv$ lifetime of dust against destruction $\approx 4 \times 10^8 \, \text{yr}$
- dust formation \approx removal+destruction $\dot{M}_{\text{dust,inj}} = M_{\text{dust}} \times (\tau_{\text{SFR}}^{-1} + \tau_{\text{dest}}^{-1})$

Production of stardust Injection of Gas and Stardust from Stellar Sources dust **Stellar Source** gas (M_{\odot}/yr) (M_{\odot}/yr) 0.33 0.0016 Planetary Nebulae ($\sim 0.3/yr$) 0.20 0.0010 RG, Red AGB, C star winds 0.06 < 0.0001? OB, WR, other warm/hot star winds SNe (1/50 yr, $\sim 10^{-2} M_{\odot}$ dust/SN?) 0.27 0.0002? 0.001 ? if $0.5M_{\odot}$ dust/SN Novae (100/yr, $10^{-7}M_{\odot}$ dust/nova?) 0.01 0.00001

 $\sim 0.9 \qquad \sim 0.003 \qquad \text{All stellar sources} \\ \sim 0.004 \qquad if \ 0.5 M_{\odot} \ dust/SN$

 $M_{\rm dust,inj} \approx 0.0035 M_{\odot} \,{\rm yr}^{-1}$

• Predicted mass of surviving stardust: $M_{\text{dust}} = \dot{M}_{\text{dust,inj}}/(\tau_{\text{SFR}}^{-1} + \tau_{\text{dest}}^{-1}) = 1.3 \times 10^{6} M_{\odot}$

• But observe

 $M_{\rm dust} \approx 0.007 \times M_{\rm ISM} = \mathbf{5} \times \mathbf{10^7 M_{\odot}}$

• Must be some other source of dust!

Implications for Grain Evolution in the ISM

- Stardust expected to account for only $\sim 3\%$ of observed dust mass.
- Bulk of dust mass must be **grown in the ISM**.
- In dense regions, time scale for atom to collide with grain surface is short:

$$\tau_{\rm acc} = \frac{1}{n_{\rm H} \sigma_{\rm dust} \Delta v} = 1 \times 10^7 \left(\frac{30 \,{\rm cm}^{-3}}{n_{\rm H}}\right) \left(\frac{{\rm km \, s}^{-1}}{\Delta v}\right) \,{\rm yr}$$

for $\sigma_{\rm dust} = 10^{-21} \,\mathrm{cm}^2/\mathrm{H}$ = geometric cross section/H

- Challenge: understanding how to form separate populations of carbonaceous material and amorphous silicates *Must be the result of UV photolysis* (see discussion in Draine 2009)
- Dust in young galaxies at high z (e.g., J1148+5251 @ z = 6.42, with $M_{\rm dust}/M_{\rm gas} \gtrsim 0.004$) can be the result of injection of small amount of stardust from SNe and high-mass stars, followed by growth in dense regions of the ISM.

Many Challenges Remain...

- The geometry of interstellar grains.
- The composition of interstellar dust both "stardust" and the materials grown in the ISM.
- The formation/destruction of PAHs
- The size distribution, and changes in the size distribution.
- The physical processes responsible for alignment of interstellar dust.
- The velocity distribution of interstellar dust in presence of MHD turbulence.
- Opacities of interstellar grain materials, from X-ray to microwave.
- Charging of interstellar grains
- Heating of gas by photoelectrons from grains
- Chemistry on grain surfaces
- Your idea here..





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